The Discharge of Two-phase Flashing Flow in a Horizontal Duct

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The well-known Lapple (1943) charts for quick graphical solutions to gas flow in pipes are extended to the case of two-phase flashing flow. (The Lapple charts were corrected by Levenspiel [1977] due to an incorrect assumption of isothermal sonic flow in the former work.) Charts for the steam-water mixture taking into account slip between phases have been presented by Moody (1966), but they cannot be used for other fluids. To the authors' knowledge, extension of the generalized design chart approach to two-phase flow has not been attempted, although methods of calculation have been outlined in Perry's (1984) handbook. The basic assumptions are homogeneous equilibrium flow (equal velocity and equal temperature in both phases) with negligible elevation changes and heat losses. Figure 1 shows the flow system and the usage of subscripts. The governing steady state conservation equations are:

$$\text{Mass} \quad G = \text{constant} \tag{1}$$

$$\text{Momentum} \quad vdP + G^2\left(vdv + \frac{4fv^2dL}{2D}\right) = 0 \tag{2}$$

$$\text{Energy} \quad h_o = h + \frac{G^2v^2}{2} \tag{3}$$

Here the specific enthalpy and specific volume are expressed in terms of quality (or vapor mass fraction) and phasic properties, i.e., $h_o = h_{fo} + x_o h_{fgo}$, $h = h_f + xh_{fg}$, and $v = v_f + xv_{fg}$. With these substitutions, the static quality is determined by solving the quadratic expression of the energy equation, Eq. 3. It can be shown that for frictionless flow ($L \rightarrow 0$), Eqs. 2 and 3 would imply an isentropic quality, i.e., $x_s = x_o + (s_{fo} - s_f)/s_{fg}$. At the other extreme with a very long duct, the kinetic energy term in Eq. 3 is necessarily small, thus yielding close to an isenthalpic quality, i.e., $x_h = x_o + (h_{fo} - h_f)/h_{fg}$. Calculated qualities in duct flow will hence be bounded by these two values.

To carry out the numerical integration, Eq. 2 is more conveniently rearranged to give

$$\Delta L = -\frac{D}{2f}\left(\frac{\Delta P}{G^2\bar{v}} + \frac{\Delta v}{\bar{v}}\right) \tag{4}$$

as suggested in Perry's handbook. Here $\bar{v}$ is the average specific volume in the incremental length ΔL. Choking is predicted to occur if for a given small ΔP, ΔL is according to Eq. 4 less than or equal to zero. Similar to Lapple's treatment, the friction factor is assumed to be constant along the duct in the numerical integration.

Scaling Parameter

Following earlier development by Epstein et al. (1983) and Grolmes and Leung (1984), a simple relation between two-phase specific volume and pressure as given by

$$\frac{v}{v_o} - 1 = \omega\left(\frac{P_o}{P} - 1\right) \tag{5}$$

was attempted by Leung (1986) to correlate flashing choked flow in a frictionless duct or nozzle. The correlating parameter ω was shown to be in the following simplified form

$$\omega = \frac{x_o v_{fgo}}{v_o} + \frac{C_{fo}T_oP_o}{v_o}\left(\frac{v_{fgo}}{h_{fgo}}\right)^2 \tag{6}$$

based on an isenthalpic (constant enthalpy) assumption. Since one is not required to obtain an exact expansion formula in such a correlation scheme (isentropic flow prevails in a frictionless duct), the above approximate result may be justified. It should be noted that ω is expressed entirely in terms of known stagnation properties, thus allowing proper dependence on quality, pressure, and specific volume variation. Furthermore, ω was demonstrated to be successful in correlating a wide range of

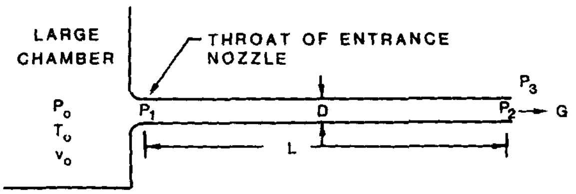

Figure 1. Two-phase flashing discharge from a large reservoir.

fluids and inlet conditions (quality from 0 to 1.0 with corresponding ω values ranging from 50 to near 1, respectively; most common fluids have ω values of about 10 for saturated liquid at 5 bar [500 kPa]).

The success of this parameter in correlating two-phase choked flow in ducts is illustrated in Figure 2 for both water (2–50 bar [200–5,000 kPa]) and nine other common fluids (all at 5 bar). Here ω is similar to the specific heat ratio k (for ideal gas) in Lapple's chart in the sense that different design curves are obtained for various ω values. The reduced mass velocity given by G/G_{max} is similar to Lapple's definition where G_{max} is now the corresponding two-phase mass velocity in a frictionless duct. The mass velocity G is evaluated numerically using the recommended method outlined above with a 20-node equal pressure decrement scheme.

Approximate Analytical Treatment

The following approximate analytical approach proves useful in correlating the above results and provides additional justification for the design chart approach. Here using the isenthalpic approximation (which is a good assumption for reasonably long ducts), Eqs. 5 and 6 are substituted into Eq. 2, the momentum equation. In essence this approximation bypasses the need to solve the energy equation simultaneously. For convenience the following dimensionless variables are defined:

$$\eta_1 \equiv P_1/P_o,\ \eta_2 \equiv P_2/P_o,\ \eta_3 \equiv P_3/P_o,$$

$$G^* \equiv G/\sqrt{P_o/v_o},\ N \equiv 4f\frac{L}{D} \quad (7)$$

The momentum equation after integration yields

$$N = 4f\frac{L}{D} = \frac{1}{G^{*2}}\left[\frac{\eta_2 - \eta_1}{1 - \omega} - \frac{\omega}{(1-\omega)^2}\ln\frac{(1-\omega)\eta_2 + \omega}{(1-\omega)\eta_1 + \omega}\right] + \ln\left[\frac{(1-\omega)\eta_2 + \omega}{(1-\omega)\eta_1 + \omega}\left(\frac{\eta_1}{\eta_2}\right)\right] \quad (8)$$

where G^* and η_1 are related via Bernoulli's equation for com-

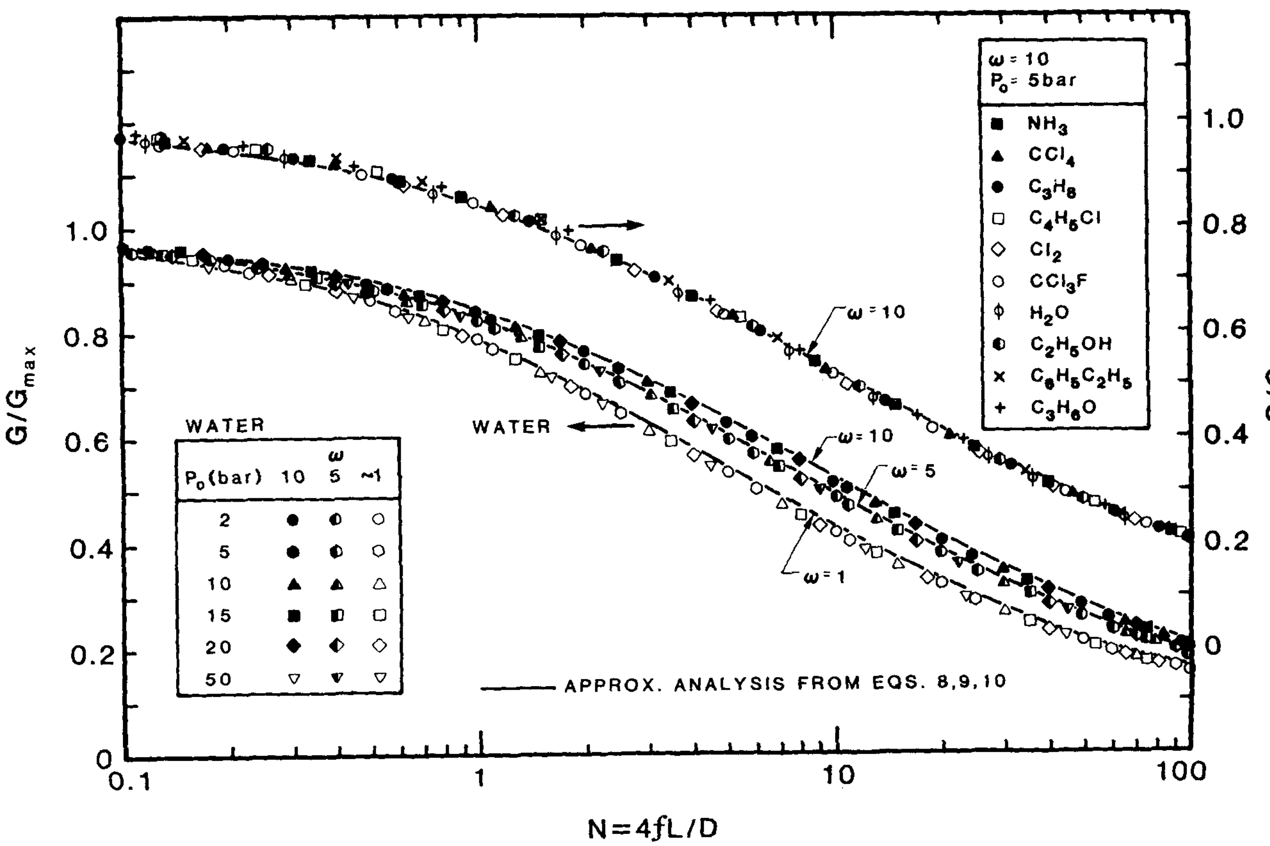

Figure 2. Correlation of critical flow in ducts based on parameter ω.

NH_3	Ammonia	CCl_3F	Trichlorofluormethane
CCl_4	Carbon tetrachloride	H_2O	Water
C_3H_8	Propane	C_2H_5OH	Ethanol
C_4H_5Cl	Chloroprene	$C_6H_5C_2H_5$	Ethyl benzene
Cl_2	Chlorine	C_3H_6O	Propylene oxide

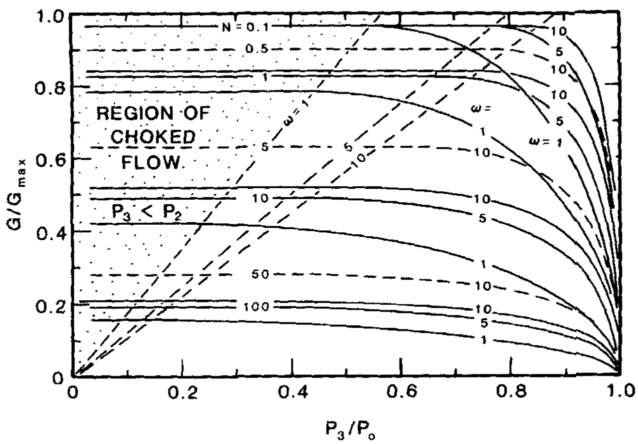

Figure 3. Design chart for two-phase flashing flow, effect of back pressure.

pressible fluid

$$G^* = \frac{\{-2[\omega \ln \eta_1 + \omega(1 - \eta_1) - (1 - \eta_1)]\}^{1/2}}{\omega\left(\frac{1 - \eta_1}{\eta_1}\right) + 1} \quad (9)$$

The exit condition can be either subcritical, in which case $\eta_2 = \eta_3$, or critical where the local choking statement would require (Grolmes and Leung, 1984)

$$G^* = \frac{\eta_2}{\sqrt{\omega}} \quad (10)$$

For the purpose of generating the appropriate curves for comparison, the following straightforward calculational scheme is followed. For given ω and η_3 values, η_1 (<1.0) is first chosen, then G^* is evaluated from Eq. 9. Next η_2 is calculated from Eq. 10 by assuming exit choking. If $\eta_2 > \eta_3$, then this assumption is valid; but if the reverse is true, then η_2 is replaced by η_3. With both η_2 and G^* known, the duct resistance factor N is evaluated from Eq. 8. To maintain the same assumption of isenthalpic flow, the critical mass velocity for frictionless duct G_{max} is given by Eq. 10 with $\eta_2 = \eta_1 = \eta$, which in turn satisfies the following transcendental equation (which results from equating the righthand sides of Eqs. 9 and 10):

$$\eta^2 + (\omega^2 - 2\omega)(1 - \eta)^2 + 2\omega^2 \ln \eta + 2\omega^2(1 - \eta) = 0 \quad (11)$$

This approximate analysis shows that for two-phase flow in ducts ω is the sole independent variable and hence is the scaling parameter once the back pressure (η_3 or P_3) is specified. For the case of exit choking with no back pressure influence, the above simple analytical treatment is shown in Figure 2 to yield remarkably good agreement with the "exact" numerical calculation. Apparently the ratio G/G_{max} is less sensitive to the assumption of the flow process as long as consistent assumptions are applied throughout. In practice, the slight discrepancies can easily be overshadowed by the uncertainty in evaluating the two-phase friction factor. In this regard, the recommended friction factor typically ranges from 0.003 (Perry, 1984) to 0.005 (Wallis, 1969).

Design Charts

The design charts shown in Figures 3 and 4 are created using water property data from the *ASME Steam Tables* (1967), but the results are applicable to other fluids as well, as demonstrated in Figure 2. Figure 4 should yield a quick graphical solution under an exit choking condition, while Figure 3 would provide a rapid assessment of the back pressure influence on the flow. After the flow reduction factor G/G_{max} is evaluated for a given N, and the corresponding G_{max} calculated using the generalized correlation by Leung (1986), the two-phase mass velocity for a given L/D duct is simply given by $G_{max}(G/G_{max})$. Also shown in Figure 4 are the curves for ideal gas with k values of 1.1 and 1.4 from Shapiro (1953). The k value of 1.1 coincides with the cur-

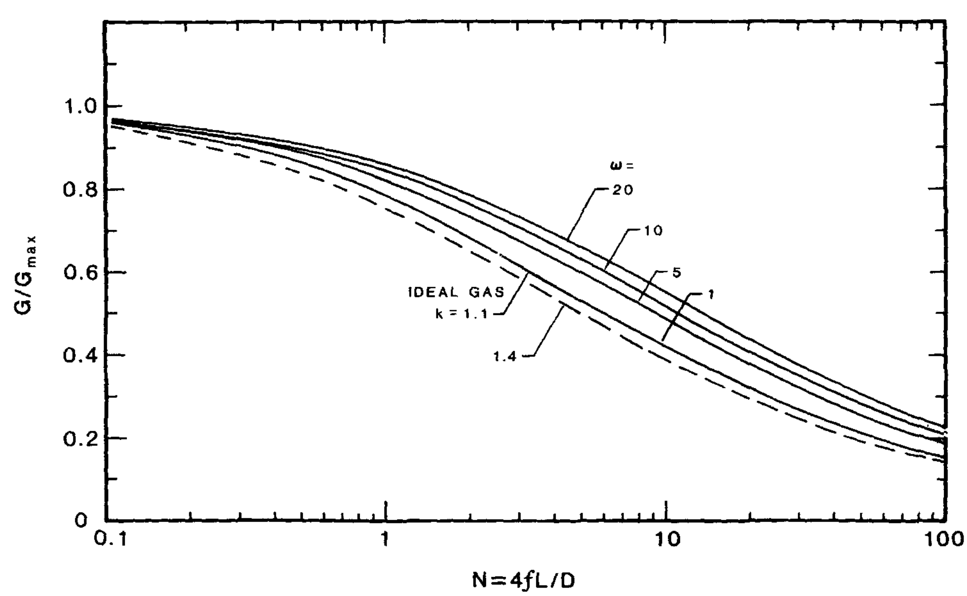

Figure 4. Design chart for two-phase flashing flow, exit choking condition.

rent analysis of $\omega = 1.0$. Finally, these charts are expected to be accurate enough for design calculations up to a reduced temperature of 0.9, for reasons discussed in an earlier paper (Leung, 1986).

Notation

C_f = liquid phase specific heat at constant pressure
D = hydraulic diameter of duct
f = Fanning friction factor
G = mass velocity or flux
G_{max} = mass velocity in a frictionless duct
h = specific enthalpy
L = duct length
N = $4fL/D$, duct resistance factor
P = pressure
s = specific entropy
T = temperature
x = quality, vapor mass fraction
ω = correlating parameters, Eq. 6
η = pressure ratio

Subscripts

f = liquid phase
fg = difference between vapor and liquid phase property
h = constant enthalpy
o = stagnation condition
s = constant entropy

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Manuscript received June 10, 1986 and revision received Aug. 26, 1986.