Calculation of Flow of Air and Diatomic Gases

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ABSTRACT

Measurement of airflow is frequently required in connection with aeronautical projects. Conventional practice, as exemplified by the *A.S.M.E. Test Code for Flow Measurement*, provides two different formulas (or sets of formulas) for the calculation of flow from pressure and temperature measurements made with standard nozzles, orifices, and the like, the proper formula for use in any particular case depending upon whether the pressure ratio across the restriction is greater or less than the critical value.

In most cases of aeronautical interest the formula for subcritical (or restricted) flow is applicable. However, the exact formula for this case is quite cumbersome, and often requires much laborious calculation. Without sacrifice of exactness, the much simpler formula for critical flow may be generalized and used in all cases merely by including a "restriction factor" that is a function of pressure ratio only, and so may be tabulated once and for all. This paper proposes such a formula and presents a table of restriction factors applicable to air and other diatomic gases.

SYMBOLS

A = throat area
C = discharge coefficient
c_p = specific heat at constant pressure
c_v = specific heat at constant volume
g = acceleration of gravity
k = ratio of specific heats ($= c_p/c_v$)
K, K' = constants (Tables 2 and 3)
N = restriction factor (Table 1)
p = absolute pressure
Δp_t = $p_{1t} - p_2$
r = pressure ratio ($= p_{1t}/p_2$)
γ = specific weight
R = gas constant
T = absolute temperature
v = specific volume
V = velocity
w = gravimetric flow
X = isentropic (or adiabatic) factor ($= r^{(k-1)/k} - 1$)

Subscripts

0 = at selected standard or reference conditions
1 = at initial, or high-side, conditions
2 = at final, or low-side, conditions
3 = at discharge, or throat, conditions (i.e., just preceding throat area A)
c = corresponding to critical pressure ratio
t = based on total pressure, total temperature

(The subscript t is added to a numerical subscript, as p_{1t}, T_{1t}, v_{1t}. Without the subscript t it is to be understood that pressures and temperatures are static values and that specific volume and specific weight are based on static pressure and temperature.)

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INTRODUCTION

MEASUREMENT OF AIRFLOW is frequently necessary in connection with tests of aircraft power plants, accessories, and the ducts required in the airplane installation. The usual method of making such measurements is by use of standard nozzles, orifices, venturi meters, or similar restrictions. From the measured pressures and temperatures the flow is calculated by well-known formulas that have been derived in many published papers and textbooks and which are summarized in reference 1. These formulas are also used to calculate the sizes of ducts and openings needed for specified flows and to determine the maximum flow capacity for given conditions.

It is customary to distinguish sharply between different types of flow of a compressible fluid. If the total pressure and temperature just preceding the restriction are constant, the flow per unit area gradually increases as the absolute pressure on the downstream side decreases, until a certain "critical" pressure ratio is reached at which it is a maximum. The flow at this pressure ratio is termed "critical." Further decrease of the downstream pressure does not change the flow, which under this condition is still termed "critical" or sometimes "supercritical." When the downstream pressure is higher than that corresponding to the critical ratio, the flow is a function of this pressure and is termed "subcritical." The words "restricted" and "nonrestricted," or "affected" and "unaffected," or "subsonic" and "supersonic" are often used instead of "subcritical" and "critical," respectively.

The exact formula for subcritical flow is quite cumbersome, and approximations have been proposed by different writers.[2, 3] However, for any group of gases having approximately the same ratio of specific heats or isentropic exponent, it is possible, by tabulation of a certain function called the "restriction factor" (perhaps "subcritical flow factor" would be a better name, but "restriction factor" is the term originally applied and now generally adopted), to use without approximation what is essentially the simple formula for critical flow over the entire range of pressure ratios. Such a table (Table 1) is given here for air and diatomic gases, which have approximately the same ratio of specific heats, taken as 1.3947 for reasons explained later in more detail. A similar table could be easily made up for other gases if the need should arise.

TABLE 1

Values of Restriction Factor N for Air and Diatomic Gases

$$w = \frac{KCAp_uN}{\sqrt{T_u}}$$

A = throat area
C = discharge coefficient
K = constant (Table 2)
N = restriction factor

p_u = high-side pressure (total)
p_l = low-side pressure (static)
T_u = high-side temperature (total)
w = gravimetric flow

To obtain volumetric flow at any specified pressure and temperature, multiply w by the specific volume at that pressure and temperature.

N is based on isentropic exponent
$k = 1.3947$, or $(k - 1)/k = 0.283$

p_u/p_l	0	1	2	3	4	5	6	7	8	9
1.0000	0	0.006540	0.009249	0.01133	0.01308	0.01462	0.01602	0.01730	0.01850	0.01962
1.0001	0.02068	0.02169	0.02265	0.02358	0.02447	0.02532	0.02616	0.02696	0.02774	0.02850
1.0002	0.02924	0.02996	0.03067	0.03136	0.03203	0.03269	0.03334	0.03397	0.03460	0.03521
1.0003	0.03581	0.03640	0.03698	0.03756	0.03812	0.03868	0.03922	0.03977	0.04030	0.04082
1.0004	0.04134	0.04186	0.04236	0.04287	0.04336	0.04385	0.04433	0.04481	0.04529	0.04576
1.0005	0.04622	0.04668	0.04713	0.04758	0.04803	0.04847	0.04891	0.04935	0.04978	0.05020
1.0006	0.05063	0.05105	0.05146	0.05187	0.05228	0.05269	0.05309	0.05349	0.05389	0.05429
1.0007	0.05468	0.05507	0.05545	0.05583	0.05621	0.05659	0.05697	0.05734	0.05771	0.05808
1.0008	0.05845	0.05881	0.05917	0.05953	0.05989	0.06024	0.06059	0.06094	0.06129	0.06164
1.0009	0.06198	0.06233	0.06267	0.06301	0.06334	0.06368	0.06401	0.06435	0.06468	0.06500
1.0010	0.06533	0.06566	0.06598	0.06630	0.06662	0.06694	0.06726	0.06757	0.06789	0.06820
1.0011	0.06851	0.06882	0.06913	0.06944	0.06974	0.07005	0.07035	0.07065	0.07095	0.07125
1.0012	0.07155	0.07185	0.07214	0.07244	0.07273	0.07302	0.07331	0.07360	0.07389	0.07418
1.0013	0.07447	0.07475	0.07503	0.07532	0.07560	0.07588	0.07616	0.07644	0.07672	0.07699
1.0014	0.07727	0.07754	0.07782	0.07809	0.07836	0.07863	0.07890	0.07917	0.07944	0.07971
1.0015	0.07997	0.08024	0.08050	0.08077	0.08103	0.08129	0.08155	0.08181	0.08207	0.08233
1.0016	0.08259	0.08284	0.08310	0.08335	0.08361	0.08386	0.08412	0.08437	0.08462	0.08487
1.0017	0.08512	0.08537	0.08562	0.08586	0.08611	0.08636	0.08660	0.08685	0.08709	0.08734
1.0018	0.08758	0.08782	0.08806	0.08830	0.08854	0.08878	0.08902	0.08926	0.08950	0.08973
1.0019	0.08997	0.09020	0.09044	0.09067	0.09091	0.09114	0.09137	0.09160	0.09184	0.09207
1.0020	0.09230	0.09253	0.09275	0.09298	0.09321	0.09344	0.09366	0.09389	0.09412	0.09434
1.0021	0.09457	0.09479	0.09501	0.09524	0.09546	0.09568	0.09590	0.09612	0.09634	0.09656
1.0022	0.09678	0.09700	0.09722	0.09744	0.09765	0.09787	0.09809	0.09830	0.09852	0.09873
1.0023	0.09895	0.09916	0.09937	0.09959	0.09980	0.1000	0.1002	0.1004	0.1006	0.1009
1.0024	0.1011	0.1013	0.1015	0.1017	0.1019	0.1021	0.1023	0.1025	0.1027	0.1029
1.0025	0.1031	0.1033	0.1035	0.1038	0.1040	0.1042	0.1044	0.1046	0.1048	0.1050
1.0026	0.1052	0.1054	0.1056	0.1058	0.1060	0.1062	0.1064	0.1066	0.1068	0.1070
1.0027	0.1072	0.1074	0.1076	0.1078	0.1079	0.1081	0.1083	0.1085	0.1087	0.1089
1.0028	0.1091	0.1093	0.1095	0.1097	0.1099	0.1101	0.1103	0.1105	0.1107	0.1108
1.0029	0.1110	0.1112	0.1114	0.1116	0.1118	0.1120	0.1122	0.1124	0.1125	0.1127
1.003	0.1129	0.1148	0.1166	0.1184	0.1202	0.1219	0.1236	0.1253	0.1270	0.1286
1.004	0.1303	0.1319	0.1334	0.1350	0.1366	0.1381	0.1396	0.1411	0.1426	0.1440
1.005	0.1455	0.1469	0.1483	0.1497	0.1511	0.1525	0.1539	0.1552	0.1566	0.1579
1.006	0.1592	0.1605	0.1618	0.1631	0.1644	0.1656	0.1669	0.1681	0.1693	0.1706
1.007	0.1718	0.1730	0.1742	0.1754	0.1765	0.1777	0.1789	0.1800	0.1812	0.1823
1.008	0.1835	0.1846	0.1857	0.1868	0.1879	0.1890	0.1901	0.1912	0.1922	0.1933
1.009	0.1944	0.1954	0.1965	0.1975	0.1986	0.1996	0.2006	0.2017	0.2027	0.2037
1.010	0.2047	0.2057	0.2067	0.2077	0.2087	0.2096	0.2106	0.2116	0.2125	0.2135
1.011	0.2145	0.2154	0.2164	0.2173	0.2182	0.2192	0.2201	0.2210	0.2219	0.2228
1.012	0.2238	0.2247	0.2256	0.2265	0.2274	0.2283	0.2291	0.2300	0.2309	0.2318
1.013	0.2327	0.2335	0.2344	0.2353	0.2361	0.2370	0.2378	0.2387	0.2395	0.2404
1.014	0.2412	0.2420	0.2429	0.2437	0.2445	0.2453	0.2462	0.2470	0.2478	0.2486
1.015	0.2494	0.2502	0.2510	0.2518	0.2526	0.2534	0.2542	0.2550	0.2558	0.2565
1.016	0.2573	0.2581	0.2589	0.2596	0.2604	0.2612	0.2619	0.2627	0.2635	0.2642
1.017	0.2650	0.2657	0.2665	0.2672	0.2680	0.2687	0.2694	0.2702	0.2709	0.2716
1.018	0.2724	0.2731	0.2738	0.2745	0.2753	0.2760	0.2767	0.2774	0.2781	0.2788
1.019	0.2795	0.2803	0.2810	0.2817	0.2824	0.2831	0.2838	0.2844	0.2851	0.2858
1.020	0.2865	0.2872	0.2879	0.2886	0.2893	0.2899	0.2906	0.2913	0.2920	0.2926
1.021	0.2933	0.2940	0.2946	0.2953	0.2960	0.2966	0.2973	0.2979	0.2986	0.2992
1.022	0.2999	0.3005	0.3012	0.3018	0.3025	0.3031	0.3038	0.3044	0.3051	0.3057
1.023	0.3063	0.3070	0.3076	0.3082	0.3088	0.3095	0.3101	0.3107	0.3114	0.3120
1.024	0.3126	0.3132	0.3138	0.3144	0.3151	0.3157	0.3163	0.3169	0.3175	0.3181
1.025	0.3187	0.3193	0.3199	0.3205	0.3211	0.3217	0.3223	0.3229	0.3235	0.3241
1.026	0.3247	0.3253	0.3259	0.3265	0.3271	0.3276	0.3282	0.3288	0.3294	0.3300
1.027	0.3305	0.3311	0.3317	0.3323	0.3329	0.3334	0.3340	0.3346	0.3351	0.3357
1.028	0.3363	0.3368	0.3374	0.3380	0.3385	0.3391	0.3397	0.3402	0.3408	0.3413
1.029	0.3419	0.3424	0.3430	0.3435	0.3441	0.3446	0.3452	0.3457	0.3463	0.3468
1.030	0.3474	0.3479	0.3485	0.3490	0.3495	0.3501	0.3506	0.3512	0.3517	0.3522
1.031	0.3528	0.3533	0.3538	0.3544	0.3549	0.3554	0.3559	0.3565	0.3570	0.3575
1.032	0.3580	0.3586	0.3591	0.3596	0.3601	0.3607	0.3612	0.3617	0.3622	0.3627
1.033	0.3632	0.3637	0.3643	0.3648	0.3653	0.3658	0.3663	0.3668	0.3673	0.3678
1.034	0.3683	0.3688	0.3693	0.3698	0.3703	0.3708	0.3713	0.3718	0.3723	0.3728
1.035	0.3733	0.3738	0.3743	0.3748	0.3753	0.3758	0.3763	0.3768	0.3773	0.3778
1.036	0.3782	0.3787	0.3792	0.3797	0.3802	0.3807	0.3812	0.3816	0.3821	0.3826
1.037	0.3831	0.3836	0.3840	0.3845	0.3850	0.3855	0.3859	0.3864	0.3869	0.3874
1.038	0.3878	0.3883	0.3888	0.3892	0.3897	0.3902	0.3906	0.3911	0.3916	0.3920
1.039	0.3925	0.3930	0.3934	0.3939	0.3944	0.3948	0.3953	0.3957	0.3962	0.3967
1.040	0.3971	0.3976	0.3980	0.3985	0.3989	0.3994	0.3998	0.4003	0.4007	0.4012
1.041	0.4016	0.4021	0.4025	0.4030	0.4034	0.4039	0.4043	0.4048	0.4052	0.4057

TABLE 1 (*Continued*)

p_u/p_t	0	1	2	3	4	5	6	7	8	9
1.042	0.4061	0.4065	0.4070	0.4074	0.4079	0.4083	0.4087	0.4092	0.4096	0.4101
1.043	0.4105	0.4109	0.4114	0.4118	0.4122	0.4127	0.4131	0.4135	0.4140	0.4144
1.044	0.4148	0.4153	0.4157	0.4161	0.4165	0.4170	0.4174	0.4178	0.4183	0.4187
1.045	0.4191	0.4195	0.4199	0.4204	0.4208	0.4212	0.4216	0.4221	0.4225	0.4229
1.046	0.4233	0.4237	0.4241	0.4246	0.4250	0.4254	0.4258	0.4262	0.4266	0.4271
1.047	0.4275	0.4279	0.4283	0.4287	0.4291	0.4295	0.4299	0.4303	0.4307	0.4312
1.048	0.4316	0.4320	0.4324	0.4328	0.4332	0.4336	0.4340	0.4344	0.4348	0.4352
1.049	0.4356	0.4360	0.4364	0.4368	0.4372	0.4376	0.4380	0.4384	0.4388	0.4392
1.05	0.4396	0.4435	0.4474	0.4512	0.4550	0.4588	0.4625	0.4661	0.4697	0.4733
1.06	0.4768	0.4803	0.4838	0.4872	0.4905	0.4939	0.4972	0.5004	0.5037	0.5069
1.07	0.5100	0.5131	0.5162	0.5193	0.5224	0.5254	0.5283	0.5313	0.5342	0.5371
1.08	0.5400	0.5428	0.5456	0.5484	0.5512	0.5539	0.5566	0.5593	0.5620	0.5646
1.09	0.5673	0.5699	0.5724	0.5750	0.5775	0.5800	0.5825	0.5850	0.5875	0.5899
1.10	0.5923	0.5947	0.5971	0.5994	0.6017	0.6041	0.6064	0.6086	0.6109	0.6131
1.11	0.6154	0.6176	0.6198	0.6220	0.6241	0.6263	0.6284	0.6305	0.6326	0.6347
1.12	0.6368	0.6388	0.6409	0.6429	0.6449	0.6469	0.6489	0.6508	0.6528	0.6547
1.13	0.6566	0.6586	0.6605	0.6624	0.6642	0.6661	0.6679	0.6698	0.6716	0.6734
1.14	0.6752	0.6770	0.6788	0.6805	0.6823	0.6840	0.6858	0.6875	0.6892	0.6909
1.15	0.6926	0.6942	0.6959	0.6976	0.6992	0.7008	0.7025	0.7041	0.7057	0.7073
1.16	0.7088	0.7104	0.7120	0.7135	0.7151	0.7166	0.7181	0.7197	0.7212	0.7227
1.17	0.7241	0.7256	0.7271	0.7286	0.7300	0.7315	0.7329	0.7343	0.7357	0.7372
1.18	0.7386	0.7399	0.7413	0.7427	0.7441	0.7454	0.7468	0.7481	0.7495	0.7508
1.19	0.7521	0.7534	0.7548	0.7561	0.7573	0.7586	0.7599	0.7612	0.7624	0.7637
1.20	0.7650	0.7662	0.7674	0.7687	0.7699	0.7711	0.7723	0.7735	0.7747	0.7759
1.21	0.7771	0.7783	0.7794	0.7806	0.7817	0.7829	0.7840	0.7852	0.7863	0.7874
1.22	0.7886	0.7897	0.7908	0.7919	0.7930	0.7941	0.7951	0.7962	0.7973	0.7984
1.23	0.7994	0.8005	0.8015	0.8026	0.8036	0.8046	0.8057	0.8067	0.8077	0.8087
1.24	0.8097	0.8107	0.8117	0.8127	0.8137	0.8147	0.8157	0.8166	0.8176	0.8186
1.25	0.8195	0.8205	0.8214	0.8223	0.8233	0.8242	0.8251	0.8261	0.8270	0.8279
1.26	0.8288	0.8297	0.8306	0.8315	0.8324	0.8333	0.8342	0.8350	0.8359	0.8368
1.27	0.8376	0.8385	0.8393	0.8402	0.8410	0.8419	0.8427	0.8436	0.8444	0.8452
1.28	0.8460	0.8468	0.8477	0.8485	0.8493	0.8501	0.8509	0.8517	0.8524	0.8532
1.29	0.8540	0.8548	0.8556	0.8563	0.8571	0.8579	0.8586	0.8594	0.8601	0.8609
1.30	0.8616	0.8623	0.8631	0.8638	0.8645	0.8653	0.8660	0.8667	0.8674	0.8681
1.31	0.8688	0.8696	0.8703	0.8710	0.8716	0.8723	0.8730	0.8737	0.8744	0.8751
1.32	0.8757	0.8764	0.8771	0.8777	0.8784	0.8791	0.8797	0.8804	0.8810	0.8817
1.33	0.8823	0.8829	0.8836	0.8842	0.8849	0.8855	0.8861	0.8867	0.8873	0.8880
1.34	0.8886	0.8892	0.8898	0.8904	0.8910	0.8916	0.8922	0.8928	0.8934	0.8940
1.35	0.8945	0.8951	0.8957	0.8963	0.8968	0.8974	0.8980	0.8985	0.8991	0.8997
1.36	0.9002	0.9008	0.9013	0.9019	0.9024	0.9030	0.9035	0.9041	0.9046	0.9051
1.37	0.9057	0.9062	0.9067	0.9072	0.9078	0.9083	0.9088	0.9093	0.9098	0.9103
1.38	0.9108	0.9113	0.9118	0.9123	0.9128	0.9133	0.9138	0.9143	0.9148	0.9153
1.39	0.9158	0.9162	0.9167	0.9172	0.9177	0.9181	0.9186	0.9191	0.9196	0.9200
1.40	0.9205	0.9209	0.9214	0.9218	0.9223	0.9227	0.9232	0.9236	0.9241	0.9245
1.41	0.9250	0.9254	0.9258	0.9263	0.9267	0.9271	0.9276	0.9280	0.9284	0.9288
1.42	0.9293	0.9297	0.9301	0.9305	0.9309	0.9313	0.9317	0.9321	0.9325	0.9329
1.43	0.9333	0.9337	0.9341	0.9345	0.9349	0.9353	0.9357	0.9361	0.9365	0.9369
1.44	0.9372	0.9376	0.9380	0.9384	0.9387	0.9391	0.9395	0.9399	0.9402	0.9406
1.45	0.9410	0.9413	0.9417	0.9420	0.9424	0.9427	0.9431	0.9434	0.9438	0.9441
1.46	0.9445	0.9448	0.9452	0.9455	0.9459	0.9462	0.9465	0.9469	0.9472	0.9475
1.47	0.9479	0.9482	0.9485	0.9489	0.9492	0.9495	0.9498	0.9501	0.9505	0.9508
1.48	0.9511	0.9514	0.9517	0.9520	0.9523	0.9526	0.9529	0.9533	0.9536	0.9539
1.49	0.9542	0.9545	0.9548	0.9550	0.9553	0.9556	0.9559	0.9562	0.9565	0.9568
1.50	0.9571	0.9574	0.9576	0.9579	0.9582	0.9585	0.9588	0.9590	0.9593	0.9596
1.51	0.9599	0.9601	0.9604	0.9607	0.9609	0.9612	0.9615	0.9617	0.9620	0.9622
1.52	0.9625	0.9628	0.9630	0.9633	0.9635	0.9638	0.9640	0.9643	0.9645	0.9648
1.53	0.9650	0.9653	0.9655	0.9658	0.9660	0.9662	0.9665	0.9667	0.9670	0.9672
1.54	0.9674	0.9677	0.9679	0.9681	0.9683	0.9686	0.9688	0.9690	0.9693	0.9695
1.55	0.9697	0.9699	0.9701	0.9704	0.9706	0.9708	0.9710	0.9712	0.9714	0.9716
1.56	0.9719	0.9721	0.9723	0.9725	0.9727	0.9729	0.9731	0.9733	0.9735	0.9737
1.57	0.9739	0.9741	0.9743	0.9745	0.9747	0.9749	0.9751	0.9753	0.9755	0.9757
1.58	0.9759	0.9760	0.9762	0.9764	0.9766	0.9768	0.9770	0.9772	0.9773	0.9775
1.59	0.9777	0.9779	0.9781	0.9782	0.9784	0.9786	0.9788	0.9789	0.9791	0.9793
1.60	0.9795	0.9796	0.9798	0.9800	0.9801	0.9803	0.9805	0.9806	0.9808	0.9809
1.61	0.9811	0.9813	0.9814	0.9816	0.9817	0.9819	0.9821	0.9822	0.9824	0.9825
1.62	0.9827	0.9828	0.9830	0.9831	0.9833	0.9834	0.9836	0.9837	0.9839	0.9840
1.63	0.9841	0.9843	0.9844	0.9846	0.9847	0.9848	0.9850	0.9851	0.9853	0.9854
1.64	0.9855	0.9857	0.9858	0.9859	0.9861	0.9862	0.9863	0.9865	0.9866	0.9867
1.65	0.9868	0.9870	0.9871	0.9872	0.9873	0.9875	0.9876	0.9877	0.9878	0.9880
1.66	0.9881	0.9882	0.9883	0.9884	0.9885	0.9887	0.9888	0.9889	0.9890	0.9891
1.67	0.9892	0.9893	0.9895	0.9896	0.9897	0.9898	0.9899	0.9900	0.9901	0.9902
1.68	0.9903	0.9904	0.9905	0.9906	0.9907	0.9908	0.9909	0.9910	0.9911	0.9912
1.69	0.9913	0.9914	0.9915	0.9916	0.9917	0.9918	0.9919	0.9920	0.9921	0.9922
1.70	0.9923	0.9924	0.9925	0.9925	0.9926	0.9927	0.9928	0.9929	0.9930	0.9931
1.71	0.9932	0.9932	0.9933	0.9934	0.9935	0.9936	0.9937	0.9937	0.9938	0.9939
1.72	0.9940	0.9940	0.9941	0.9942	0.9943	0.9944	0.9944	0.9945	0.9946	0.9947
1.73	0.9947	0.9948	0.9949	0.9949	0.9950	0.9951	0.9952	0.9952	0.9953	0.9954

(*Continued on page 312*)

TABLE 1 (*Continued*)

p_{1t}/p_2	0	1	2	3	4	5	6	7	8	9
1.74	0.9954	0.9955	0.9956	0.9956	0.9957	0.9957	0.9958	0.9959	0.9959	0.9960
1.75	0.9961	0.9961	0.9962	0.9962	0.9963	0.9964	0.9964	0.9965	0.9965	0.9966
1.76	0.9966	0.9967	0.9968	0.9968	0.9969	0.9969	0.9970	0.9970	0.9971	0.9971
1.77	0.9972	0.9972	0.9973	0.9973	0.9974	0.9974	0.9975	0.9975	0.9976	0.9976
1.78	0.9977	0.9977	0.9978	0.9978	0.9978	0.9979	0.9979	0.9980	0.9980	0.9981
1.79	0.9981	0.9981	0.9982	0.9982	0.9983	0.9983	0.9983	0.9984	0.9984	0.9984
1.80	0.9985	0.9985	0.9985	0.9986	0.9986	0.9986	0.9987	0.9987	0.9987	0.9988
1.81	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990	0.9990	0.9990	0.9991
1.82	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993	0.9993	0.9993
1.83	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995	0.9995	0.9995
1.84	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997	0.9997	0.9997	0.9997
1.85	0.9997	0.9997	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998
1.86	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
1.87	0.9999	0.9999	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

$N = 1.000$ for all higher pressure ratios.

PROPOSED FORMULA

In place of separate formulas for subcritical and critical flow, it is proposed to substitute the single general formula

$$w = KCAp_{1t}N/\sqrt{T_{1t}}$$

which is merely the formula for critical flow multiplied by N, the tabulated restriction factor. The value of N varies from zero to unity for subcritical flow and is unity for critical flow.

This formula was introduced into the General Electric Company some years ago by the late V. Petrovsky and has been found to be convenient and timesaving for both air and steam calculations. Values of N may be read from curves such as are shown in Fig. 1. These are reasonably satisfactory but have the usual disadvantages attendant upon purely graphical presentation. The formula is used so frequently and the advantages of a table over a chart are so manifest that an accurate table for air and diatomic gases has been prepared, based, for the sake of consistency, on fundamental constants which are essentially those adopted in the A.S.M.E. codes for flow measurement[1] and for the testing of air and gas compressors.[4] These codes have been extensively used in the aeronautical industry, and it appears best to standardize on values already in current use.

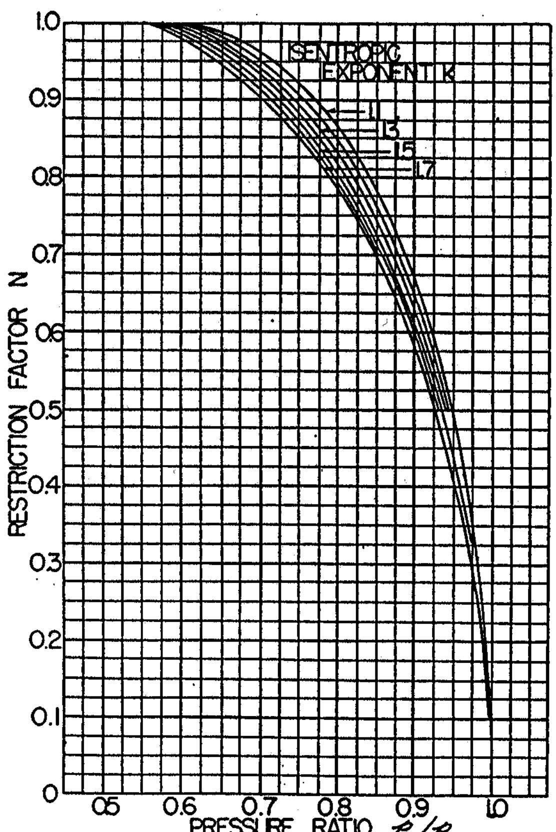

FIG. 1. Values of restriction factor N for different values of isentropic exponent k.

DERIVATION OF FORMULA

The general derivation of the flow formula is well known, and the details will not be emphasized here. The case considered is that of the flow of air or of a diatomic gas through a nozzle or orifice of any kind with a velocity increase corresponding to the pressure drop. The pressure ratio is taken as the high-side *total* pressure divided by the low-side *static* pressure, and this is obviously always greater than unity if there is any flow. The term "high-side" refers to entrance conditions and is somewhat more expressive than the words "inlet" or "initial" frequently used. The use of total pressure and total temperature at this section makes unnecessary the introduction of any approach-velocity correction. The usual assumption as to uniformity of flow over the cross section is made.

The low-side static pressure is the pressure of the region into which the fluid is discharged. For subcritical flow, this is the same as the pressure at area A (or throat), but for critical flow it may be, and usually is, less. Since it is only in the subcritical case that the low-side pressure affects the flow, it is not important in general (at least from the standpoint of flow measurement) to know the actual low-side pressure if it is known definitely that it is below the critical value.

Considering first subcritical flow, the theoretical velocity head acquired in an expansion from inlet to discharge pressure is equal to the change of enthalpy (otherwise called available energy, isentropic heat drop, or adiabatic heat drop), which is equal to the specific heat at constant pressure multiplied by the isentropic temperature drop.

$$V_3^2/2g = c_p T_{1t} X/(X+1)$$

or

$$V_3 = \sqrt{2gc_p T_{1t} X/(X+1)} \qquad (1)$$

The flow per unit of effective area is

$$w/CA = V_3/v_3 \qquad (2)$$

Assuming, as usual, an isentropic process (pv^k = constant),

$$v_3 = v_{1t} r^{1/k} = v_{1t}(X+1)^{1/(k-1)} \qquad (3)$$

so that

$$\frac{w}{CA} = \sqrt{\frac{2gc_p T_{1t}}{v_{1t}^2} \frac{X}{(X+1)^{(k+1)/(k-1)}}} \qquad (4)$$

It is apparent that w/CA is a maximum when the function

$$\phi(X) = X/(X+1)^{(k+1)/(k-1)} \qquad (5)$$

is a maximum. Differentiating this expression with respect to X and equating the derivative to zero, the critical value of X is found to be

$$X_c = (k-1)/2 \qquad (6)$$

For the critical pressure ratio, then,

$$\frac{w_c}{CA} = \sqrt{\frac{2gc_p T_{1t}}{v_{1t}^2} \frac{X_c}{(X_c+1)^{(k+1)/(k-1)}}} \qquad (7)$$

Dividing Eq. (4) by Eq. (7), the formula for the restriction factor is obtained.

$$N = \frac{w}{w_c} = \sqrt{\frac{X}{X_c}\left[\frac{(X_c+1)}{(X+1)}\right]^{(k+1)/(k-1)}} \qquad (8)$$

For a constant value of k, this expression is a function of X only. It may therefore be tabulated for different values of r.

The flow for any pressure ratio may then be calculated from the formula

$$\frac{w}{CA} = N\sqrt{\frac{2gc_p T_{1t}}{v_{1t}^2} \frac{X_c}{(X_c+1)^{(k+1)/(k-1)}}} \qquad (9)$$

which is reduced to the customary simple form of the critical flow formula by using the relation

$$c_p - c_v = R$$

or

$$c_p[1 - (1/k)] = p_{1t} v_{1t}/T_{1t}$$

so that

$$v_{1t} = (c_p T_{1t}/p_{1t})[(k-1)/k] \qquad (10)$$

TABLE 2

Values of K for Use in General Flow Formula

p_{1t}	T_{1t}	A	w	K
Pounds per sq.in. absolute	°F. absolute	sq.in.	lbs. per sec.	0.53033
			lbs. per min.	31.820
			lbs. per hour	1909.2
		sq.ft.	lbs. per sec.	76.368
			lbs. per min.	4582.1
			lbs. per hour	274,920.
	°C. absolute	sq.in.	lbs. per sec.	0.39529
			lbs. per min.	23.717
			lbs. per hour	1423.0
		sq.ft.	lbs. per sec.	56.921
			lbs. per min.	3415.3
			lbs. per hour	204,920.
Inches Hg absolute	°F. absolute	sq.in.	lbs. per sec.	0.26048
			lbs. per min.	15.629
			lbs. per hour	937.74
		sq.ft.	lbs. per sec.	37.510
			lbs. per min.	2250.6
			lbs. per hour	135,030.
	°C. absolute	sq.in.	lbs. per sec.	0.19415
			lbs. per min.	11.649
			lbs. per hour	698.95
		sq.ft.	lbs. per sec.	27.958
			lbs. per min.	1677.5
			lbs. per hour	100,650.
Pounds per sq.ft. absolute	°F. absolute	sq.in.	lbs. per sec.	0.0036829
			lbs. per min.	0.22097
			lbs. per hour	13.258
		sq.ft.	lbs. per sec.	0.53033
			lbs. per min.	31.820
			lbs. per hour	1909.2
	°C. absolute	sq.in.	lbs. per sec.	0.0027450
			lbs. per min.	0.16470
			lbs. per hour	9.8821
		sq.ft.	lbs. per sec.	0.39529
			lbs. per min.	23.717
			lbs. per hour	1423.0

Substituting Eqs. (6) and (10) in Eq. (9), the formula becomes

$$w/CA = Kp_{1t}N/\sqrt{T_{1t}} \qquad (11)$$

with

$$K = \left[\frac{k^2}{k-1}\left(\frac{2}{k+1}\right)^{(k+1)/(k-1)}\right]^{1/2}\sqrt{\frac{g}{c_p}} \qquad (12)$$

For $k = 1.39470$,

$$K = 1.2855\sqrt{g/c_p}$$

and

$$w/CA = 1.2855\sqrt{(g/c_p)}(p_{1t}N/\sqrt{T_{1t}}) \qquad (13)$$

which is the general formula for flow permitting the use of any consistent units.

It is probably desirable to assume some particular system of units and reduce the formula still further in order to eliminate as far as possible the amount of calculation required in routine computations. There is, however, no general agreement on standard units to be used for w, A, p, and T, and Table 2 has been prepared to give the value of K corresponding to all the different combinations of units likely to be used in practice. The working formula for flow for any given values of C, A, p_{1t}, T_{1t}, and r then becomes

$$w = KCAp_{1t}N/\sqrt{T_{1t}}$$

with N taken from Table 1 and K from Table 2.

Selection of Basic Constants

The basic physical constants used in preparing the tables are essentially those adopted in A.S.M.E. test codes for measurement of flow and for compressor testing. In particular, the following constant values have been used directly or are implied.

p_0 = 14.70 lbs. per sq.in. absolute
T_0 = 527.6°F. absolute (68°F.)
γ_0 = 0.075 lb. per cu.ft. (exact value by definition)
$(k - 1)/k$ = 0.283 (exact value by definition)
k = 1.39470
c_p = 0.242967 B.t.u. per lb. per °F.
g = 32.1740 ft. per sec. per sec.

The "long" values of k and c_p were used in computing the constant K in order to ensure exact consistency with the selected value of $(k - 1)/k$, although even here, where the computation needs to be made only once and for all, this is a rather academic proceeding. Normally, these two values would be rounded off to three or four significant figures.

The reasons for selection of these constants are explained in the A.S.M.E. codes and in the references there given. Since it may not be convenient in all cases to consult these references, the reasons will be briefly summarized here.

First, "Normal Air" is defined as air with an average amount of CO_2 and sufficient moisture so that its specific weight at the selected standard conditions—p_0 = 14.70 lbs. per sq.in. absolute and T_0 = 527.6°F. absolute (68°F.)—is exactly γ_0 = 0.075 lb. per cu.ft. These values were selected chiefly because they (or values substantially equal to them) were already being used as standards in allied fields, especially in heating and ventilating engineering.

By combination of the two fundamental thermodynamic relations

$$c_p - c_v = c_p[1 - (1/k)] = R \tag{14}$$

and

$$pv = RT \tag{15}$$

the relation connecting c_p, k, and v (or γ) is obtained

$$c_p[1 - (1/k)] = pv/T = p/\gamma T \tag{16}$$

Investigation of published data on the specific heat of air at temperatures of usual interest (0–200°F.), and containing about the same amount of moisture as Normal Air, led to the selection of 0.243 B.t.u. per lb. per °F. as a reasonable average value. Substituting this in Eq. (16) and using on the right-hand side the values applicable to Normal Air, it was found that

$$(k - 1)/k = 1 - (1/k) = 0.28296$$

Since $(k - 1)/k$ is a function more commonly used in thermodynamic calculations than k itself and since the value derived from Eq. (16) can be only an average value at best, this figure was rounded off to 0.283, which for the purpose of establishing a consistent set of values for thermodynamic calculations was taken as exact. The corresponding value of c_p then became 0.242967, which would be rounded off to 0.243 for usual calculations but must be retained in full when exact consistency is required. Similarly, the corresponding consistent value of the ratio of specific heats is k = 1.39470.

The ratio $(k - 1)/k$ is selected as a basic value because many important thermodynamic formulas, including formulas for the flow of compressible fluids,[5] may be expressed in terms of a certain function

$$(p_1/p_2)^{(k-1)/k} - 1$$

which is sometimes known as "adiabatic factor" but is probably better designated as "isentropic" factor. Tables of this function are now available in various publications, as well as in A.S.M.E. codes. It has been extensively used in the aeronautical industry, usually being denoted by the symbol, Y, which was originally given to it.[5] With increasing use, however, the American Standards Association has decreed that, in order to avoid certain conflicts, it shall be denoted by the symbol X rather than Y, and it is so designated in most recent publications. The symbol X is therefore adopted in this paper.

Use of Formula

The formula is given in terms of p_{1t} and T_{1t}, the total pressure and temperature, respectively, on the upstream side of the restriction. In most cases the reading of the usual thermometer or thermocouple is nearly enough equal to the total temperature, but for extremely high velocities the reading is usually intermediate between the true static and the true total temperature and a correction must be applied. This is true, of course, no matter what formula is used.

The total pressure is the pressure that would be shown by an impact tube pointing directly upstream. Sometimes a static rather than a total pressure is measured. The static pressure and temperature may be used in the formula if the usual correction for velocity of approach is made. This correction is often negligibly small.

In making flow measurements it is usually considered better practice to measure the differential

$$\Delta p_t = p_{1t} - p_2$$

directly, together with either one or both of the gage pressures, rather than to depend only on the latter. The reduction to common units and calculation of the pressure ratio is combined into one operation by the formula

$$\frac{p_{1t}}{p_2} = 1 + \frac{K'\Delta p_t}{p_2} = \frac{1}{1 - (K'\Delta p_t/p_{1t})}$$

Values of K' are given in Table 3 for the cases most likely to occur. The constants there given assume that, in accordance with usual practice, the readings of mercury columns have been reduced to equivalent heights at 32°F. and that the readings of water columns have been reduced to, or are considered closely enough equal to, equivalent heights at 60°F.

The formula gives what is called "gravimetric" flow. This term indicates merely a flow that the practical engineer is accustomed to express in pounds per second, pounds per minute, or the like and takes no stand on the controversial issue as to whether it should be denoted "mass" flow or "weight" flow. The corresponding volumetric flow at any specified pressure and temperature is obtained by multiplying by the specific volume at that pressure and temperature.

In certain problems the formula is used in the form

$$N = (\sqrt{T_{1t}}w/A)/KCp_{1t}$$

as, for example, when checking the capacity of a restriction or duct for a specified flow. If the calculated value of N is greater than unity, it is impossible to pass the specified flow with the specified pressure and temperature. With N exactly equal to unity, the velocity is sonic. A value of N less than unity indicates the fraction of the total possible capacity being utilized.

Calculation of Tables

Most of the values of N have been calculated using seven-place logarithmic tables and a calculating machine. For the low-pressure ratios, where the number of significant figures given in seven-place logarithmic tables is rather small, the 16-place table of natural logarithms prepared by the Federal Works Agency under the sponsorship of the National Bureau of Standards[6] was used. In some cases the logarithms were themselves calculated directly from the appropriate infinite series.

Since the function N changes rapidly at low-pressure ratios, the increments of pressure ratio have been chosen with a view to making the differences between the tabulated values small enough for convenient interpolation throughout the entire table. However, interpolation should not be necessary in most practical cases. The table was originally prepared with the values of N carried out to the nearest fifth significant figure, which for the greater portion of the table is equivalent to five decimal places. However, it seemed that this number of places was greater than was needed in the great majority of practical cases and, in fact, would seldom be justifiable when the accuracy of the usual test and other data available are taken into consideration. The values here given have therefore been rounded off to the nearest fourth significant figure, with the incidental advantage of facilitating interpolation if this is required.

TABLE 3

Values of K' for Use in Calculating Pressure Ratio

$$p_{1t}/p_2 = 1 + (K'\Delta p_t/p_2) = \frac{1}{1 - (K'\Delta p_t/p_{1t})}$$

p_{1t}, p_2	Δp_t	K'
Inches Hg absolute*	Inches H_2O†	0.07347
	Inches Hg*	1.000
	Pounds per sq.in.	2.036
Pounds per sq.in. absolute	Inches H_2O†	0.03609
	Inches Hg*	0.4912
	Pounds per sq.in.	1.000

* At 32°F.
† At 60°F.

It is shown in reference 2 (formula D) that for low-pressure ratios the flow is closely proportional to

$$\sqrt{p_2\Delta p_t}$$

Equating this closely approximate formula to the formula here used and solving for N, it is found that for low-pressure ratios N is proportional to

$$\sqrt{p_2\Delta p_t}/p_{1t}$$

If all pressures are expressed in the same units, the constant of proportionality varies from 2.06806 at a pressure ratio of unity to 2.06416 at a ratio of 1.0500, with intermediate values nearly in direct proportion. Occasionally, this relation may be of use in calculating values of N not tabulated directly. Since N is a function of pressure ratio only, any convenient values of absolute pressure may be assumed in making the calculation.

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[2] Moss, S. A., *Measurement of Flow of Air and Gas with Nozzles,* Trans. A.S.M.E., Applied Mechanics, Vol. 50, paper 3, 1928.

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[6] *Table of Natural Logarithms,* Vol. III, prepared by Federal Works Agency, Work Projects Administration for the City of New York, sponsored by and obtainable from the National Bureau of Standards, Washington, D.C.