Easily Size Relief Devices and Piping for Two-Phase Flow

A unified approach is given for flashing and nonflashing two-phase flow. Design charts are used to solve complex equations, making the method simple to use.

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This article will provide practicing engineers with a unified methodology for sizing relief devices — both rupture disks (RDs) and pressure relief valves (PRVs) — and associated relief piping for two-phase flow. Sizing methods for gases and for nonflashing liquids (liquid below its atmospheric boiling point) have been standard for over three decades. Sizing methods for two-phase flow, in particular flashing flow, have been lacking and are often misunderstood. The American Petroleum Institute (API) method for flashing flow *(1)* is probably the most widely used in the chemical process industries (CPI), but it lacks both a sound theoretical basis and validated experimental data. Despite its origin dating back to the 1950s and its popularity over the years, this method is regarded as obsolete and can lead to undersizing of relief devices *(2)*.

The present approach is based on the classical homogeneous equilibrium model (HEM) that assumes equal velocities (or no slip) and thermal equilibrium between both phases. HEM is favored by AIChE's Design Institute for Emergency Relief Systems (DIERS) for emergency relief sizing design *(3,4)*.

Furthermore, the flow capacity curve for saturated water discharge in the current ASME pressure vessel code *(5)* is in close agreement with the HEM prediction *(2)*. For pressure relief sizing, use of HEM yields a slightly conservative design.

This article will:

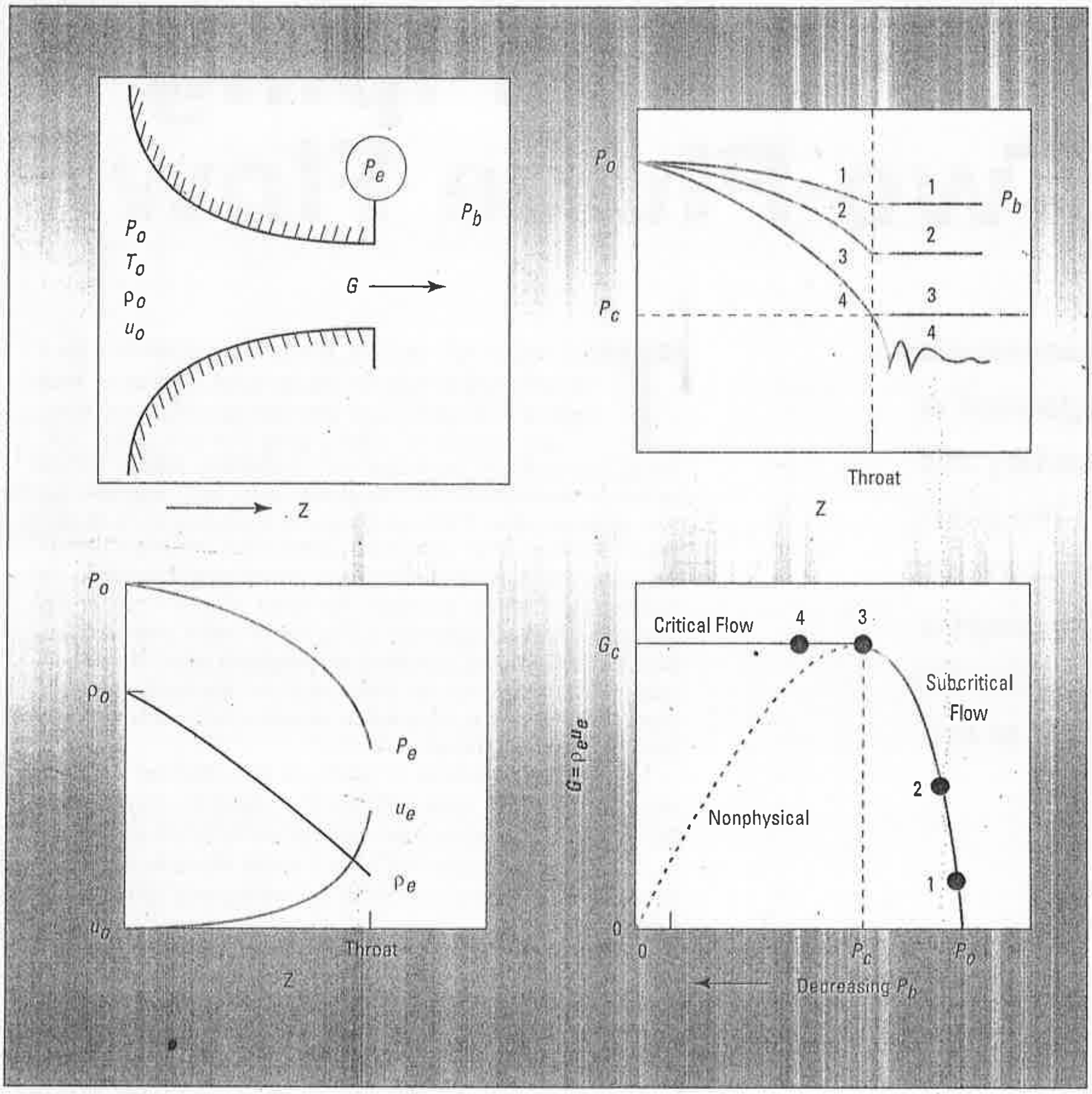
• Present a unified treatment for sizing PRVs and RDs in two-phase service, covering flashing as well as nonflashing two-phase flow.

• Emphasize the pencil-and-paper approach for a better understanding of the theory and method, although the underlying equations and formulae can be easily programmed into handheld calculators or laptop computers.

• Provide applications and worked examples to illustrate the sizing methodology.

• Reduce to the API gas-sizing and liquid-sizing methods in the limits.

Note that the first sidebar contains terms used in this article; the second details common units and conversions.



■ *Figure 1. Operation of a simple nozzle at various back pressures.*

Critical flow behavior

If a compressible fluid (gas, saturated liquid, or two-phase) is allowed to expand across a nozzle, or the end of a constant-diameter pipe, its velocity and specific volume will increase with continuing pressure drop. Figure 1 provides graphs for a nozzle.

For a given set of upstream conditions, the mass flux (flow rate per unit area) G will increase until a limiting velocity is reached in the throat (for a nozzle) or at the exit end (for a pipe). It can be shown that the limiting velocity is the local velocity of sound in the flowing fluid. The flow rate that corresponds to the limiting velocity is known as the critical (or choked) flow rate. The absolute pressure ratio of the pressure in the throat at sonic velocity P_c to the inlet pressure P_o is called the critical (choking) pressure ratio. P_c is known as the critical (or choking) flow pressure. Further reduction in the back pressure below P_c would have no effect on the flow rate. (See Point 4 in Figure 1.)

API/ASME SINGLE-PHASE FLOW FORMULAE

PRV sizing for gas flow

The API formulae are given in U.S. engineering units (lb/h, in.², psia, °R) as follows:

Critical flow condition:

$$\frac{W}{A} = CK_dK_bP_o\left(\frac{M_w}{T_oZ}\right)^{0.5} \qquad (1)$$

Subcritical flow condition:

$$\frac{W}{A} = 735F_2K_d\left[\frac{M_wP_o(P_o - P_b)}{T_oZ}\right]^{0.5} \qquad (2)$$

Terminology

Design Institute for Emergency Relief System (DIERS): institute under the auspices of AIChE, founded to investigate requirements for vent lines in the case of two-phase venting.

Set Pressure: the inlet pressure at which the relief device is set to open (burst).

Overpressure: a pressure increase over the set pressure of the relief device, usually expressed as a percentage of gage set pressure.

Stagnation Pressure: the pressure that would be observed if a flowing fluid were brought to rest along an isentropic path.

Back Pressure: the static pressure existing at the outlet of a pressure relief device as a result of the pressure in the discharge system. It is the sum of the superimposed and built-up back pressures.

Built-Up Back Pressure: pressure existing at the outlet of a pressure relief device caused by flow through that particular device into a discharge system.

Superimposed Back Pressure: the static pressure existing at the outlet of a pressure relief device at the time the device is required to operate. It is the result of pressure in the discharge system from other sources.

Units and Conversions

One difficulty often encountered in relief sizing is units and conversions. In this article, SI units are recommended. Here is a quick conversion table:

Parameter	U.S. Engineering Unit		SI Unit
P	1 psi	=	6,897 N/m² or Pa
V	1 ft³	=	2.8317 × 10⁻² m³
m_o	1 lb	=	0.4536 kg
C_p	1 Btu/lb•°F	=	4,187 J/kg•K
ρ	1 lb/ft³	=	16.02 kg/m³
h_{fg}	1 Btu/lb	=	2,326.1 J/kg
W	1 lb/s	=	0.4536 kg/s
G	1 lb/s•ft²	=	4.878 kg/m²•s
A	1 in.²	=	1/1,550 m²
v	1 ft³/lb	=	0.06243 m³/kg
L	1 ft	=	0.305 m

In addition the following constants are noted:

Gravitational acceleration	g	=	9.81 m/s²
Gas constant	R	=	8,314 Pa-m³/K•kg-mole

Also worth noting are:

1 Pa = 1 N/m² = 1 kg/m•s²
1 J = 1 N-m

The groupings $G/\sqrt{P_o\rho_o}$ and $\rho_o C_p T_o P_o \left(\frac{v_{fgo}}{h_{fgo}}\right)^2$ in the ω parameter are used frequently — note that they are dimensionless. (This is left as an exercise for readers.)

where:

$$F_2 = \left[\left(\frac{k}{k-1}\right)\left(\frac{P_b}{P_o}\right)^{\frac{2}{k}}\left(\frac{1-\left(P_b/P_o\right)^{(k-1)/k}}{1-\left(P_b/P_o\right)}\right)\right]^{0.5} \quad (3)$$

and:

$$C = 520\left[k\left(\frac{2}{k+1}\right)^{(k+1)/(k-1)}\right]^{0.5} \quad (4)$$

The critical flow pressure ratio is:

$$\frac{P_c}{P_o} = \left(\frac{2}{k+1}\right)^{k/(k-1)} \quad (5)$$

The corresponding formulae in SI units (kg/s, m², N/m², K) are:

Critical flow condition:

$$\frac{W}{A} = K_d K_b P_o\left(\frac{M_w}{RT_oZ}\right)^{1/2}\left[k\left(\frac{2}{k+1}\right)^{(k+1)/(k-1)}\right]^{1/2} \quad (6)$$

Subcritical flow condition:

$$\frac{W}{A} = K_d P_o\left(\frac{M_w}{RT_oZ}\right)^{1/2}\left\{\left(\frac{2k}{k-1}\right)\left[\left(\frac{P_b}{P_o}\right)^{2/k}-\left(\frac{P_b}{P_o}\right)^{(k+1)/k}\right]\right\}^{1/2} \quad (7)$$

These formulae are typically found in fluid mechanics textbooks with a value of unity for K_d, K_b, and Z. Note, however, that the compressibility Z is arbitrarily entered into these API formulae since they are derived with the assumption of $Z = 1$ (ideal gas).

PRV sizing for liquid flow

The API formula for liquid service can be written as:

$$Q = 38K_d K_w K_v A\left(\frac{P_o - P_b}{\gamma}\right)^{0.5} \quad (8)$$

where Q is in gal/min, A = in.², P_o = psia, and γ is the specific gravity of the liquid. The equivalent formulae written is SI units (m³/s, m², N/m², kg/m³) are:

$$Q = K_d K_w K_v A\left(\frac{2\left(P_o - P_b\right)}{\rho_f}\right)^{0.5} \quad (9)$$

or, in terms of mass flow (kg/s):

$$\frac{W}{A} = K_d K_w K_v\left[2\rho_f\left(P_o - P_b\right)\right]^{0.5} \quad (10)$$

With the exception of the various flow coefficients, one can readily recognize that these are the two different forms of the familiar Bernoulli equations.

Pressure drop in piping for gas flow

The API equation for gas flow in a constant-diameter pipe makes use of the isothermal flow approximation due to Lapple *(6)*:

$$4f\frac{L}{D} = \frac{1}{M_2^2}\left(\frac{P_1}{P_2}\right)^2\left[1-\left(\frac{P_2}{P_1}\right)^2\right] - 2\ln\left(\frac{P_1}{P_2}\right) \qquad (11)$$

where the exit Mach number is given in terms of U.S. engineering units (lb/h, psia, °R, in.):

$$M_2 = 1.702 \times 10^{-5}\left(\frac{W}{P_2 D^2}\right)\left(\frac{ZT}{M_w}\right)^{1/2} \qquad (12)$$

Equation 11 can be easily rewritten in terms of flow rate and in SI units (N/m², kg/m³, kg/m²s):

$$4f\frac{L}{D} = \frac{P_1\rho_1}{G^2}\left[1-\left(\frac{P_2}{P_1}\right)^2\right] - 2\ln\left(\frac{P_1}{P_2}\right) \qquad (13)$$

where $\rho_1 = P_1 M_w/(RT_1)$ and $G = W/A$.

EFFECT OF TWO-PHASE FLOW ON RELIEF CAPACITY

Simply stated, sizing pressure relief systems requires achieving a balance between the volumetric generation rate and the volumetric discharge rate through the relief device. However, the determination of the volumetric generation rate is beyond the scope of this article. API RP-520 discusses the required relief capacities for a number of common contingencies *(1)*. Recent DIERS technology focuses on the runaway reaction contingency and its associated venting requirement *(7)*.

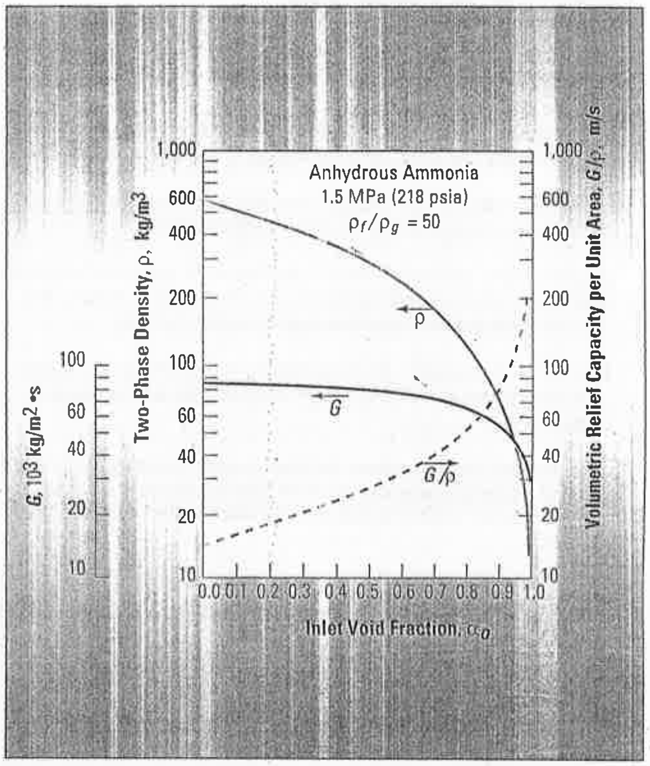
In equation form, the volumetric balance can be written as:

$$\frac{K_d A G}{\rho} = Q_{vol} \qquad (14)$$

where Q_{vol} is on a volumetric basis.

For a given relief device and relieving pressure, the effect of two-phase flow is to increase the mass-flow rate and decrease the volumetric flow rate relative to that for all vapor/gas flow. Using ammonia properties at 1.5 MPa (218 psia), Figure 2 shows that both the choked flow mass flux G and the fluid density ρ decrease with increasing void fraction α_o entering the relief device. In contrast, the volumetric flow term G/ρ (this is actually a volumetric flux term) increases rapidly with increasing void fraction.

In this illustration, the volumetric flow per unit area is about 10 times larger for all-vapor venting ($\alpha_o = 1.0$) than for two-phase venting with $\alpha_o = 0.3$. This would translate into a 10 times larger vent area for the two-phase venting case vs. that for vapor venting.



■ *Figure 2. Illustration of variation of G, ρ, and G/ρ with α_o.*

HEM TWO-PHASE FLOW METHOD

DIERS technology demonstrates that the traditional HEM flow is the most conservative model for estimating flow capacity from PRVs and RDs *(7)*. In this HEM formulation, the two-phase flow mixture is treated much like the classical compressible gas, while undergoing an adiabatic expansion with equal velocities and temperatures in both phases. For a flashing system, this would imply thermodynamic vapor-liquid equilibrium (VLE). For a nonflashing system (*i.e.*, no mass transfer takes place between the two phases), thermal equilibrium means equal temperatures in both phases. The other extreme for nonflashing flow is thermally insulated (or frozen) flow (*i.e.*, no heat transfer takes place between the phases). Fortunately, for most practical cases the resulting flow rates do not differ much (less than 10%) between these two limiting assumptions for nonflashing flow.

In flow passages greater than 0.1 m in length (frequently encountered in relief systems), the HEM model, in fact, provides a best-estimate calculation for a flashing system.

Many two-phase flow models have appeared in the literature. Most of these incorporate thermodynamic nonequilibrium or mechanical nonequilibrium (slip between phases); however, invariably they are empirical in nature, and not easily extrapolated to other conditions and different fluids.

HEM for nozzles

For flow in a frictionless nozzle, the HEM flow rate requires seeking a maximum in the mass flux G expression as derived from the energy equation:

$$\text{Maximize: } G = \frac{\left[2(h_o - h)\right]^{0.5}}{v} \quad (15)$$

where, for a flashing system:

$h_o = h_{fo} + x_o h_{fgo}$ (stagnation enthalpy)
$h = h_f + x h_{fg}$ (local enthalpy)
$v = v_f + x v_{fg}$ (local specific volume)
x_o = inlet vapor mass fraction (quality)
x = local vapor mass fraction (quality).

(Note that consistent units are required; SI units are recommended.)

The local quality during expansion is evaluated along an isentropic (constant entropy) line:

$$x = \frac{S_o - S_f}{S_{fg}} = \frac{S_{fo} + x_o S_{fgo} - S_f}{S_{fg}} \quad (16)$$

The above procedure, while not complicated, is tedious and requires extensive thermodynamic properties. Critical flow corresponds to the maximum of the above G expression (see Figure 1). Note that the dashed curve to the left of the maximum in Figure 1 represents a nonphysical region, since further lowering the back pressure has no effect on the flow.

HEM for pipes

For pipe flow, HEM requires solving the equations of conservation of mass, energy, and momentum. The momentum equation is in differential form which requires partitioning the pipe into segments and carrying out numerical integration. For constant-diameter pipe, we have:

Mass:

$$G = \text{constant} \quad (17)$$

Energy:

$$h_o = h + \frac{G^2 v^2}{2} \quad (18)$$

Momentum:

$$v dP + G^2\left(v dv + \frac{4 f v^2 dL}{2D}\right) + g \cos\theta \, dL = 0 \quad (19)$$

Equation 19, the momentum equation, can be solved more conveniently by rewriting it and solving it numerically *(8)*:

$$\Delta L = -\frac{\bar{v}\Delta P + G^2 \bar{v}\Delta v}{\frac{2f}{D} G^2 \bar{v}^2 + g \cos\theta} \quad (20)$$

where $\bar{v}$ is the average specific volume in the pressure increment ΔP, and Δv is the incremental specific volume over ΔP.

In the numerical integration of Eq. 20 for a given pipe length L, the following steps are suggested *(8)* (note that detailed thermodynamic properties are required also):

Step Action

1. G is known or guessed.

2. Increments of pressure are taken from the initial to the final pressure.

3. $\bar{v}$ and Δv are obtained for each increment for a constant-enthalpy process.

4. ΔL for each ΔP taken is computed from Eq. 20.

5. Total length of pipe L is $\Sigma\Delta L$.

6. If ΔL is negative, then ΔP is too large.

7. A critical flow condition corresponds to $\Delta L = 0$, and the final pressure corresponds to choked pressure.

8. If $\Sigma\Delta L > L$, then G was guessed too small, and Steps 1–7 are repeated with a larger G. If $\Sigma\Delta L < L$, then G was guessed too large; Steps 1-7 are repeated with a smaller G.

9. A converged solution is obtained when $\Sigma\Delta L = L$ to within a given tolerance.

This method is presented to show how long and tedious are typical calculations. In contrast, the method that will be presented here is easy to use via equations and charts.

TWO-PHASE FLOW VIA INTEGRAL EQUATIONS

DIERS methodology *(7)* makes use of the integral form of the variable-area momentum balance for the nozzle flow, and the constant-area momentum balance for the pipe flow. This greatly reduces the complexity of the calculations when used in conjunction with an appropriate two-phase expansion law, *i.e.*, a P-v relationship.

Nozzle flow

Since the nozzle is adiabatic and frictionless, flow is isentropic, and the Gibbs equation becomes:

$$T\,dS = 0 = dh - v\,dP \quad (21)$$

Substituting for $(h_o - h)$ in the expression for G (Eq. 15) yields the integral equation:

$$G = \left[2\int_{P_o}^{P} -v\,dP\right]^{1/2} / v \quad (22)$$

where the integration is done from the stagnation pressure P_o to the throat pressure P.

For an ideal gas, the well-known isentropic expansion law is:

$$P\,v^k = P_o\,v_o^{\,k} \quad (23)$$

Nomenclature

- a,b = parameters in Eq. 25
- A = area
- A_{in} = inlet pipe flow area
- A_n = nozzle area
- c = sonic velocity
- C = as defined by Eq. 4
- C_p = (liquid) specific heat at constant pressure
- C_v = specific heat at constant volume
- D = diameter
- f = Fanning friction factor
- F = derating factor in Eq. 52
- Fi = flow inclination number
- g = gravitational acceleration
- G = mass flux or velocity
- G^* = $G/\sqrt{P_o \rho_o}$
- G_E = equilibrium mass flux
- G_{NE} = nonequilibrium mass flux
- G_o = mass flux corresponding to nozzle
- h = enthalpy
- h_{fg} = latent heat of vaporization
- H = height
- k = isentropic exponent
- K_b = capacity correction factor due to back pressure for gas service valve
- K_d = discharge coefficient of PRV nozzle
- K_l = individual fitting loss coefficient
- K_{in} = combined flow resistance in inlet pipe
- K_v = correction factor due to viscosity
- K_w = correction factor due to back pressure for liquid service valve
- L = pipe length
- L_E = equilibrium flow length
- M = Mach number
- M_w = molecular weight
- N_{NE} = as defined by Eq. 71
- P = absolute pressure
- ΔP_{in} = pressure drop for inlet pipe
- Q = volumetric rate
- R = gas constant
- S = entropy
- T = absolute temperature
- T_R = thrust force
- u = velocity
- v = specific volume
- $\bar{v}$ = average specific volume in pressure increment ΔP
- $\bar{v}_{in}$ = average two-phase specific volume in the inlet pipe section
- W = mass-flow rate
- x = quality or vapor mass fraction
- Z = compressibility

Greek letters

- α = void fraction or vapor volume fraction
- γ = liquid specific gravity
- ρ = fluid density
- η = P/P_o pressure ratio
- θ = angle of inclination with the vertical

Subscripts

- a = ambient
- act = actual
- b = back pressure
- c = critical or choked
- e = exit
- E = equilibrium
- f = liquid
- fg = difference between vapor and liquid properties
- g = vapor or gas
- in = inlet
- max = maximum
- min = minimum
- n = nozzle
- NE = nonequilibrium
- o = stagnation inlet condition, or outlet for nozzle
- req = required
- s = saturated condition, also set condition
- st = saturation at transition in Eq. 65
- v = vapor
- vol = volumetric basis
- 1 = pipe inlet
- 2 = pipe exit

Substituting into Eq. 22 and carrying out the integration yields the subcritical flow equation (similar to Eq. 7) for G. The maximum for G can be found by setting $dG/dP = 0$ and solving for the critical pressure ratio:

$$\frac{P_c}{P_o^*} = \left(\frac{2}{k+1}\right)^{k/(k-1)} \qquad (24)$$

For two-phase flashing flow, the P-v relation should be given by a constant entropy flash calculation. However, in practice, the result is almost indistinguishable when a constant enthalpy (isenthalpic) flash calculation is used; the latter seems to be much more common with commercially available flash routines.

DIERS uses a general two-phase P-v relation given by:

$$\frac{v}{v_o} - 1 = a\left(\frac{P_o}{P} - 1\right) + b\left(\frac{P_o}{P} - 1\right)^2 \qquad (25)$$

The P-v data generated from flash calculations are used to find the best fit for the two parameters a and b. Practically speaking, only two flash calculations at two lower pressures are needed to solve for a and b. Typically b is much smaller than a. When $b = 0$ and $a = 1$, one recovers the relation $Pv = P_o v_o$ for the ideal gas with $k = 1.001$. (API recommends a value of 1.001 to avoid mathematical problems when dividing by $k-1$.) When $b = 0$ and $a = 0$, the equation yields $v = v_o$, the incompressible liquid state.

Pipe flow

In fully turbulent flow where the friction factor can be considered relatively constant, the momentum equation for a constant diameter pipe, Eq. 19, can be recast in an integral form:

$$4f\frac{L}{D} = \int \frac{-v\,dP - G^2 v\,dv}{\frac{1}{2}G^2v^2 + \frac{gD}{4f}\cos\theta} \quad (26)$$

where the integration is from the front end of the pipe (P_1, v_1) to its exit (P_2, v_2). For adiabatic pipe flow, the energy equation would provide the P-v relation at a given G value, *i.e.*, Eq. 18. As an example, for the case of an ideal gas, Eq. 18 readily yields the following P-v relation inside of the pipe:

$$Pv = P_1 v_1 \left\{1 - \left(\frac{k-1}{2k}\right)\frac{G^2 v_1}{P_1}\left[\left(\frac{v}{v_1}\right)^2 - 1\right]\right\} \quad (27)$$

where the reference is taken to be Point 1 (the beginning of the pipe). Although exact pipe flow solutions have been obtained with an ideal gas based on this expansion law *(9)*, it has been a common practice to ignore the kinetic energy term, thus reducing the formula to $Pv = P_1 v_1$. Such an isothermal flow approximation has been used frequently in design calculations for gas pipe flow, with acceptable accuracy and ease of use. The same approximation has been recommended by DIERS for a two-phase system, *i.e.*, ignoring the kinetic energy term in the energy equation and simply basing calculations on an isenthalpic flash.

ω TWO-PHASE FLOW METHODOLOGY

The so-called ω method, as developed by the author and co-workers in a number of publications *(10–15)*, simplified the classical HEM two-phase flow methodology to a set of analytical, but algebraic, equations. Some of these equations require iterative solutions. However, convenient design charts are presented to provide quick design calculations. Key features of this method are:

• This methodology is applicable to both flashing as well as nonflashing two-phase flow systems; moreover, it reduces exactly to the vapor/gas flow solution ($k = 1.001$) and the liquid flow result at its limits.

• Both nozzle and pipe configurations can be handled in a similar fashion.

• Most important of all, this method reduces to a number of key dimensionless parameters, and emphasizes the importance of physical property groups solely at inlet conditions.

• The ω method is based on a general equation of state (EOS) for describing a two-phase fluid expansion. It has some theoretical bases; however, similar to the perfect gas assumption, it is only an ideal (simplistic) approximation. Its merit lies in the utility of the results generated.

An EOS for two-phase fluid expansion is written as:

$$\frac{v}{v_o} = \omega\left(\frac{P_o}{P} - 1\right) + 1 \quad (28a)$$

$$\frac{\rho_o}{\rho} = \omega\left(\frac{P_o}{P} - 1\right) + 1 \quad (28b)$$

This form is essentially a simplification of the DIERS two-parameter EOS version of Eq. 25, *i.e.*, $a = \omega$ and $b = 0$. However, in this method ω considers physical property groups as follows:

Flashing system:

$$\omega = \frac{x_o v_{go}}{v_o} + \frac{C_p T_o P_o}{v_o}\left(\frac{v_{fgo}}{h_{fgo}}\right)^2 \quad (29a)$$

$$\omega = \alpha_o + (1 - \alpha_o)\rho_{fo} C_p T_o P_o \left(\frac{v_{fgo}}{h_{fgo}}\right)^2 \quad (29b)$$

Nonflashing system:

$$\omega = \alpha_o \quad (30a)$$

and in a more recent development:

$$\omega = \alpha_o\left(1 - 2\frac{P_o v_{fgo}}{h_{fgo}}\right) + \frac{C_p T_o P_o}{v_o}\left(\frac{v_{fgo}}{h_{fgo}}\right)^2 \quad (29c)$$

is shown to be applicable to a flashing flow system and:

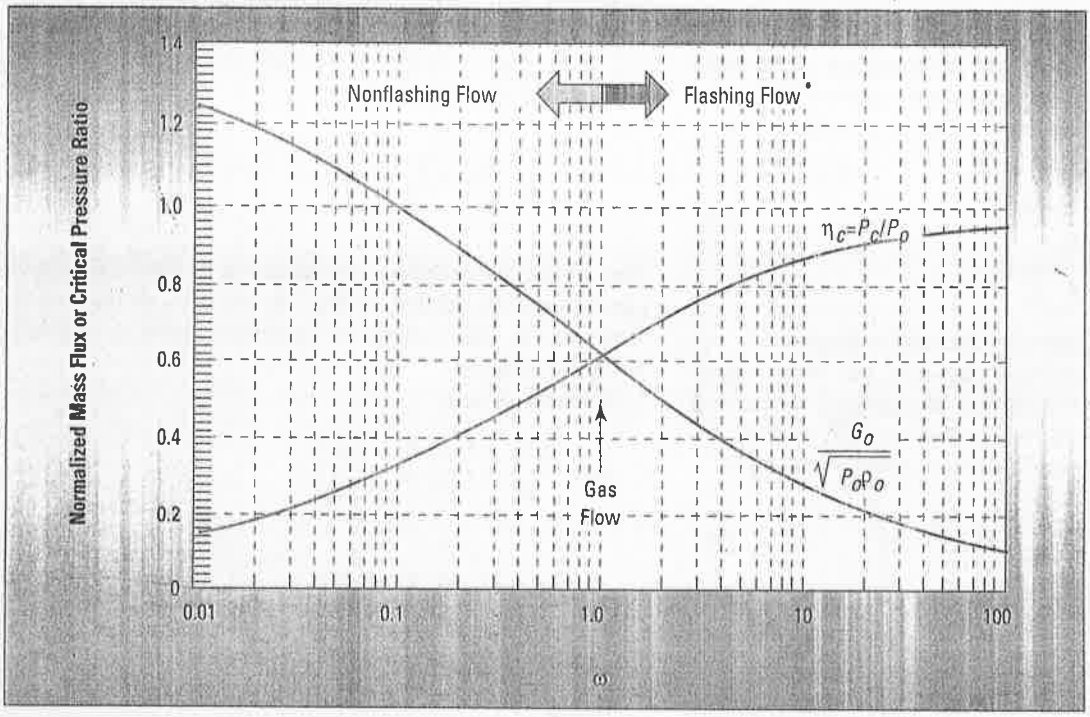
$$\omega = \frac{\alpha_o}{k} \quad (30b)$$

is suggested for nonflashing flow systems.

This method is well defined for a single-component fluid, but is not as accurate for multicomponent systems, particularly when a composition change during expansion is significant. More will be discussed on this point in later on.

Physical meaning of ω

According to Eq. 29b, ω is made up of two distinct terms: the first reflects the compressibility of the two-phase mixture due to the existing vapor/gas volume, and the second concerns the compressibility due to phase change upon depressurization or flashing. The larger the value of ω, the more volume expansion or compressible behavior the mixture exhibits.



■ *Figure 3. Generalized chart for two-phase flow through a nozzle.*

For flashing two-phase flow systems, the second term dominates until α_o approaches unity (all-vapor inlet). For nonflashing two-phase flow systems (such as air-water or nitrogen-oil), the second term vanishes (no phase change) and ω simply becomes α_o. This result allows the developing of general solutions for flashing flow to be extended without modification to nonflashing flow *(14)*. Thus, depending on the ω value, we can have the following flow characterizations:

Flashing flow	$\omega > 1$
Gas/vapor flow	$\omega \approx 1$
Nonflashing flow	$0 < \omega < 1$ $(\omega = \alpha_o$ or $\alpha_o/k)$
Liquid flow	$\omega = 0$ $(\alpha_o = 0)$

ω nozzle flow

A generalized expression for flow through a perfect nozzle is given by:

$$\frac{G}{\sqrt{P_o/v_o}} = \frac{G}{\sqrt{P_o\rho_o}} = \frac{\left\{-2\left[\omega \ln\frac{P}{P_o} + (\omega - 1)\left(1 - \frac{P}{P_o}\right)\right]\right\}^{0.5}}{\omega\left(\frac{P_o}{P} - 1\right) + 1} \qquad (31)$$

Choking or critical flow, as defined by the maximum value of G of the above equation, yields the following (transcendental) equation for the critical pressure ratio P_c/P_o:

$$\left(\frac{P_c}{P_o}\right)^2 + (\omega^2 - 2\omega)\left(1 - \frac{P_c}{P_o}\right)^2 + 2\omega^2 \ln\frac{P_c}{P_o} + 2\omega^2\left(1 - \frac{P_c}{P_o}\right) = 0 \qquad (32)$$

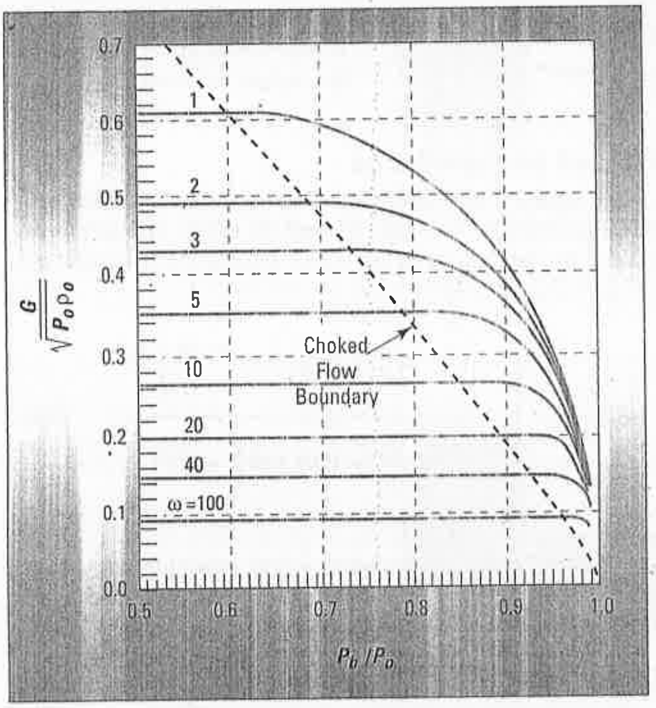
When P_c/P_o is substituted for P/P_o in Eq. 31, the critical mass flux G_c is obtained; but a much simpler formula for G_c is:

$$\frac{G_c}{\sqrt{P_o/v_o}} = \frac{P_c/P_o}{\sqrt{\omega}} \qquad (33)$$

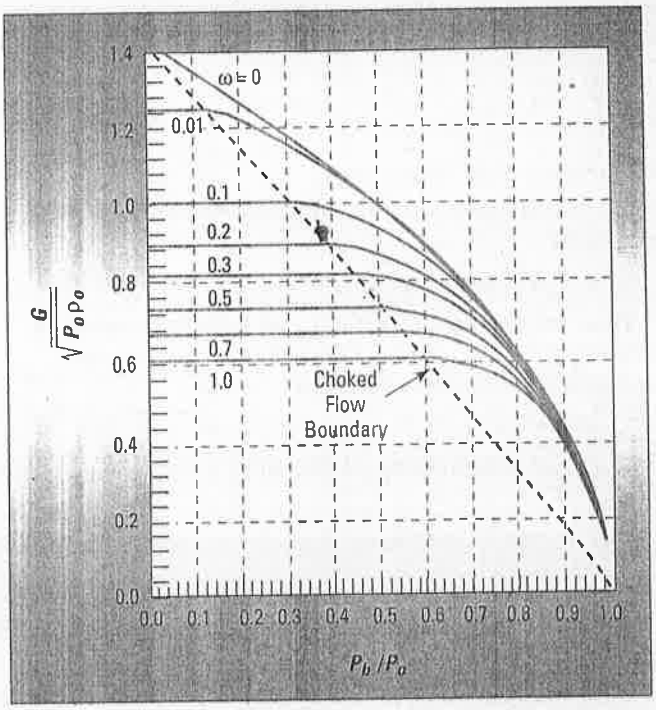
Note that Eq. 31 is a more general formula, also valid in the subcritical flow condition. Thus, if $P_c > P_b$, flow is choked and Eqs. 32 and 33 apply. Otherwise, $P_c < P_b$ and flow is subcritical (or unchoked), and P is equated to P_b in Eq. 31 to obtain the unchoked mass flux G.

The above choked flow solutions are represented by Figure 3, covering the entire two-phase flow spectrum — flashing flow lies to the right of $\omega = 1$, and nonflashing flow lies to the left of it. At $\omega = 1$, the graph recovers the classical gas nozzle-flow result ($k = 1.001$). It can also be demonstrated by setting $\omega = 1$ in Eq. 31:

$$\frac{G}{\sqrt{P_o\rho_{go}}} = \frac{P}{P_o}\left(-2\ln\frac{P}{P_o}\right)^{0.5} \qquad (34)$$



■ *Figure 4. Effect of back pressure on flashing flow through a nozzle.*



■ *Figure 5. Effect of back pressure on nonflashing flow through a nozzle.*

which yields a maximum at:

$$\frac{P_c}{P_o} = 0.606 \tag{35}$$

$$\frac{G}{\sqrt{P_o \rho_{go}}} = 0.606 \tag{36}$$

These results agree with the API formula with $k = 1.001$. (The square root term involving k in Eq. 4 yields a value of 0.606 with $k = 1.001$.) At the other limit when $\omega = 0$, *i.e.*, $\alpha_o = 0$ or when there is nonflashing liquid flow, Eq. 31 reduces to:

$$\frac{G}{\sqrt{P_o \rho_{fo}}} = \left[2\left(1 - \frac{P}{P_o}\right)\right]^{0.5} \tag{37}$$

which is the classical Bernoulli equation for incompressible flow and is also the underlying formula for the the API PRV sizing equation for liquid service.

Figures 4 and 5 illustrate the solutions to Eqs. 31 and 32 for flashing and nonflashing flow, respectively, and, in particular, show the effect of back pressure. The locus of the choking conditions is illustrated in these figures as a dashed line.

ω pipe flow equations

Pipe flow is significantly more complex than nozzle flow. Here, the pressure drop is due to acceleration (sometimes known as momentum), friction in the pipe, and gravity (elevation change). The last two components are not encountered in nozzle flow. Here, our discussion will be limited to the turbulent regime where the Fanning friction factor f can be assumed to be constant or some suitable average value. For a two-phase flow application, a Fanning turbulent friction factor of 0.005 is adequate for most cases. The ω method reveals three dimensionless parameters governing such turbulent two-phase pipe flow:

ω = the same definition as for nozzle flow

$$4f\frac{L}{D} = \left(4f\frac{L}{D}\right)_{\text{straight pipe}} + \Sigma K_i \tag{38}$$

$$Fi = \frac{\rho_o g L \cos\theta}{\left(4f\frac{L}{D}\right)P_o} = \frac{\rho_o g H}{\left(4f\frac{L}{D}\right)P_o} \tag{39}$$

The fitting loss coefficients are to be included in the total piping resistance factor $4fL/D$ as shown. Here a so-called "flow inclination" number Fi is introduced for two-phase flow with height H (or $L \cos \theta$) denoting the elevation relative to the pipe inlet. Horizontal pipe flow yields $Fi = 0$ and upflow gives a positive Fi number. So, in pipe discharge the mass flux can be uniquely characterized by the following:

$$\frac{G}{\sqrt{P_o \rho_o}} = \text{function}\,(\omega, 4fL/D, Fi, P_b/P_o) \tag{40}$$

Solution:

A_n = (178 kg/s)/(12,000 kg/m²•s) = 0.0148 m² = 23 in.²

$G/G_o = A_n/A_p$ = 23 in.²/50 in.² = 0.46

Based on ω = 3.4, Fi = 0, the G/G_o chart (Figure 6) yields $4fL/D$ = 12, which is the maximum allowable value. Hence the maximum equivalent length is:

$L_{max} = 12D/4f$ = (12)(0.203 m)/[(4)(0.005)] = 122 m (400 ft)

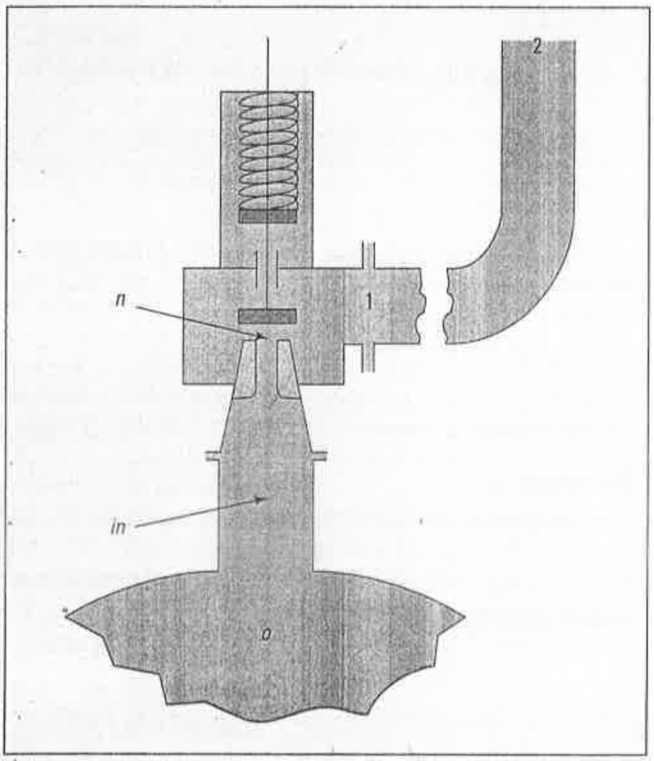
PRV SIZING, INCLUDING INLET AND OUTLET PIPING

Sizing PRV nozzles

For a required relief vent rate W_{req} (which is usually obtained from a volumetric balance consideration) and PRV inlet conditions, the minimum nozzle (or orifice) flow area is calculated by:

$$A_{min} = \frac{W_{req}}{K_d K_b G_o} \qquad (51)$$

where G_o is the discharge mass flux for a perfect nozzle geometry. Since the valve should be fully open at 10% overpressure, it is customary to evaluate G_o at this value. (In practice, the opening pressure of full-lift PRV in gas/vapor service ranges from 1–5% above the set pressure.) Furthermore, the capacity correction factor K_b due to back pressure is given for an overpressure of 10% and higher. A coefficient of discharge of 0.975 can be used for a gas service PRV when a particular valve does not have a certified value.



■ *Figure 11. PRV schematic: use of subscripts.*

Example 3

Using the same inlet conditions for ethylene as in Example 1, determine the PRV orifice size needed for a required vent rate of 180 kg/s.

In Example 1, we obtained G_o = 12,000 kg/m²•s at an inlet pressure of 2.037 MPa (295 psia) and a quality (vapor mass fraction) of 0.0091. Since this is evaluated at 10% overpressure, the PRV set pressure would be 1.86 MPa (255 psig).

Note that the flow through the valve is choked since the critical flow pressure ratio for ω = 3.4 is 0.75, yielding a choking pressure of 1.53 MPa (221 psia). Setting K_b = 1 for the time being, Eq. 51 yields A_{min} = (180)/(0.975 × 12,000) = 0.0154 m² (23.9 in.²). The next available orifice is a T with an area of 26.0 in.² (0.0168 m²). It is important to recognize that, for subsequent sizing of inlet and outlet piping, as well as for downstream equipment, the actual discharge rate based on the PRV orifice area should be used. Hence:

$$W_{act} = \frac{K_d}{F} A_n G_o \qquad (52)$$

$$= (0.975)/(0.9) \times (0.0168)(12{,}000) = 218 \text{ kg/s}$$

Note that F = 0.9. This is a derating factor and is recommended if the API nominal orifice areas are used. On the other hand, if actual orifice areas are used, F can be set equal to 1.0.

Sizing inlet piping

A PRV should be installed with as short as possible inlet piping to minimize the inlet pressure drop. See Figure 11 for a typical PRV installation and notation. API recommends to limit the inlet nonrecoverable losses to 3% or less of the (gage) set pressure *(1)*. The inlet pressure loss can be estimated by:

$$\Delta P_{in} = \frac{1}{2} K_{in} \left(\frac{W}{A_{in}} \right)^2 v_{in} \qquad (53)$$

where K_{in} is the combined entrance, rupture disk, and straight pipe resistances, A_{in} is the inlet pipe flow area, and v_{in} is the average two-phase specific volume in the inlet pipe section.

A conservative estimate would be to evaluate v_{in} at the PRV inlet pressure, but this involves a tedious calculation. A first approximation is to assume $v_{in} = v_o$, *i.e.*, at the entrance to the inlet piping. (For flashing flow with ω > 10, this approximation is, however, not so accurate.) This is essentially the same approach used by Cox and Weirick *(18)* for the gas flow case. The inlet pressure loss can be normalized by the stagnation inlet pressure at 10% overpressure:

$$\frac{\Delta P_{in}}{P_o} = \frac{K_{in}}{2}\left(\frac{K_d G^*}{F}\right)^2\left(\frac{A_n}{A_{in}}\right)^2 \quad (54)$$

To relate this ratio to the 3% rule, which by convention is based on gage set pressure, we write the following relationships:

$$\Delta P_{in} = 0.03\ (P_s - P_b) \quad (55)$$

$$P_o = 1.1\ (P_s - P_b) + P_b \quad (56)$$

By adhering to the API 3% rule, Eq. 54 can be rearranged to yield the allowable K_{in} as:

$$K_{in} = \frac{2\left(\dfrac{F}{K_d G^*}\right)^2}{\left(\dfrac{A_n}{A_{in}}\right)^2}\left[\frac{0.03\left(1-\dfrac{P_b}{P_s}\right)}{1.1\left(1-\dfrac{P_b}{P_s}\right)+\dfrac{P_b}{P_s}}\right] \quad (57)$$

Thus, for a given P_b/P_s, ω, and valve size, the maximum inlet piping resistance K_{in} can be calculated in a straightforward manner for the same size inlet piping as the valve inlet connection. For a different pipe size, the piping resistance can be corrected by multiplying by the diameter ratio to the fourth power.

A design chart is produced in Figure 12 *(19)*. Relief valve sizes appear in the figure. Here, the maximum allowable K_{in} is given as a function of the ω parameter, the orifice to inlet-pipe diameter ratio, and the ambient pressure ratio P_b/P_s. A few observations can be made from Figure 12:

- For (equilibrium) flashing discharge (ω > 1), the allowable K_{in} value is significantly higher than for the vapor flow case (ω ~ 1).
- For nonflashing (nonequilibrium) discharge (ω < 1), the allowable K_{in} is further reduced relative to the vapor case.
- For vapor discharge (ω ~ 1), the present curve yields results consistent with Cox and Weirick's findings *(19)*.
- For large valves such as types P, R, and T, the allowable K_{in} is quite restrictive. (In fact, it is common practice to employ a one size larger inlet pipe than the PRV body inlet flange because of this limitation.)

Example 4

Continuing with the ethylene example, estimate the pressure loss for an 8-in. inlet pipe (A_{in} = 50 in.² = 0.0323 m²):

$$\Delta P_{in} = \frac{K_{in}}{2}\left(\frac{218}{0.0323}\right)^2 0.00251 = 57{,}170 K_{in}$$

and for $\Delta P_{in} \le 0.03\ (P_s - P_b)$ = 52,800, P_a = 7.65 psi, and the maximum allowable K_{in} is:

K_{in} = 52,800/57,170 = 0.92

Deducting for an entrance loss coefficient of 0.5 (square cut entrance):

$4f(L_{max}/D)$ = 0.9 – 0.5 = 0.4

L_{max} = [(0.4)(8/12 ft)]/[(4)(0.005)] = 13 ft (4 m)

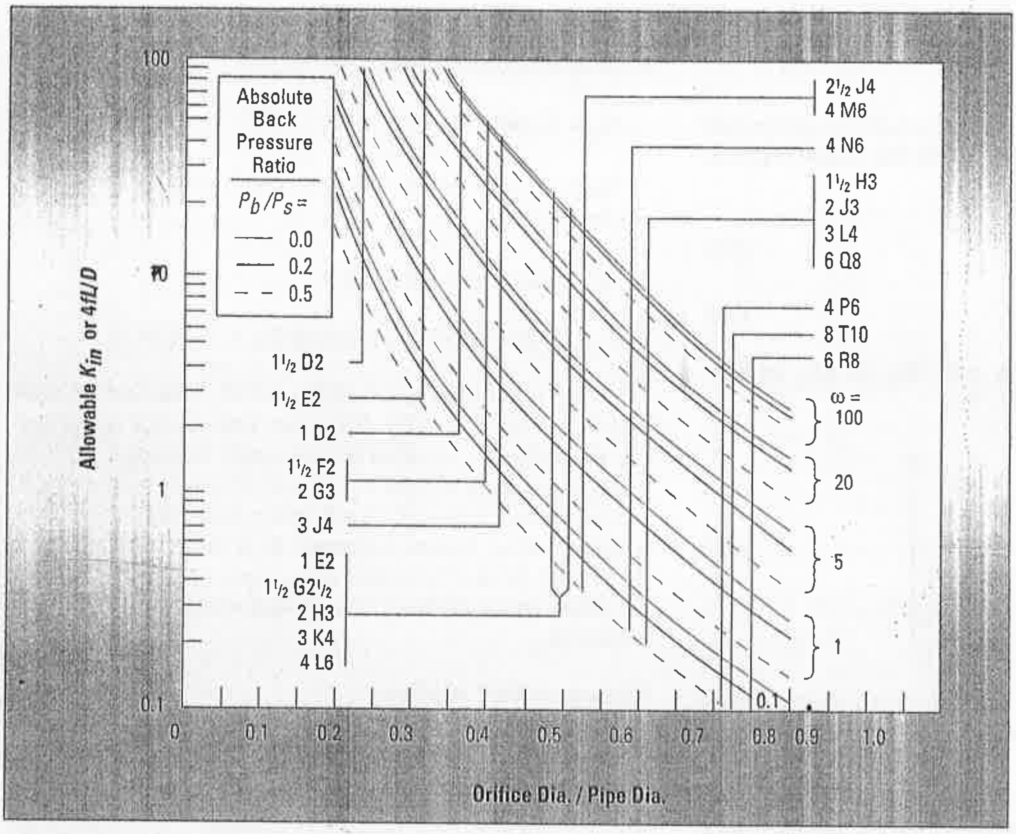
Using the design chart of Figure 12, based on ω = 3.5 and P_b/P_s ≈ 0, K_{in} ≈ 0.9. However, it should be noted that the allowable K_{in} is about 0.5 for vapor discharge (ω ~ 1), implying that for a square cut inlet, the next pipe size (10 in.) has to be employed. For flashing flow systems (ω > 1), a conservative design approach is to base size on vapor discharge. While for nonflashing systems (ω < 1), a conservative approach is to base design on nonflashing liquid discharge.

Sizing outlet piping

The limitation here is the length of the outlet pipe run, given the maximum allowable back pressure for the particular valve. The latter is specified by the valve manufacturer and is typically 10% of the set pressure for conventional (unbalanced) valves. Higher builtup back pressure would affect the valve lift, causing rapid deterioration of relief capacity. The longest equivalent pipe length that can pass the actual flow at a given pipe diameter is derived from the momentum equation:

$$4f\frac{L}{D} = 2\frac{P_o\rho_o}{(W/A_2)^2}\left\{\frac{\eta_1-\eta_2}{1-\omega}+\frac{\omega}{(1-\omega)^2}\times \ln\left[\frac{(1-\omega)\eta_2+\omega}{(1-\omega)\eta_1+\omega}\right]\right\} - 2\ln\left[\frac{(1-\omega)\eta_2+\omega}{(1-\omega)\eta_1+\omega}\left(\frac{\eta_1}{\eta_2}\right)\right] \quad (58)$$

(Recall that $\rho_o = 1/v_o$, and A_2 = outlet pipe flow area.) Note that this equation is applicable to horizontal pipe flow. However, the equation should be adequate for flashing flow even with an elevation rise in the tailpipe. This is because the fluid density is significantly reduced in the outlet pipe, resulting in negligible hydrostatic head. For nonflashing flow, the elevation rise in the outlet pipe can be detrimental and should be avoided.



■ ***Figure 12. Allowable inlet piping resistance factor for satisfying the API 3% rule.***

Referring to Figure 11, $\eta_1 = P_1/P_o$ and $\eta_2 = P_2/P_o$ denote the pressure ratios at the front end and at the exit end of the pipe, respectively. In estimating the allowable $4fL/D$ from Eq. 58, P_1 would assume the maximum allowable builtup back pressure value while P_2 is assigned the higher of either the back pressure P_b or the critical exit choking pressure:

$$P_{2c} = \frac{W}{A_2}\left(\frac{P_o \omega}{\rho_o}\right)^{1/2} \tag{59}$$

Note that this equation is another form of Eq. 43.

PRV permissible back pressure

For conventional-type PRVs (in which the bonnet is vented to the discharge side of the valve), the sum of builtup and superimposed back pressures is typically restricted to 10% of the set pressure when operating at an overpressure of 10%. At this overpressure, higher back pressures may reduce the valve capacity and induce the valve to chatter. The performance against back pressure can be improved by operating at a higher overpressure; the valve manufacturer should be consulted.

Balanced-type PRVs are designed so that the opening and closing forces due to back pressure acting on the seated disc are balanced. These valves behave better against builtup back pressure. At 10% overpressure, the sum of builtup and superimposed back pressures should not exceed 30–50%; again the valve manufacturer should be consulted. Likewise, the valve performance against back pressure can be improved by operating at a high overpressure, but, at present, available data are limited to a maximum value of 20% overpressure.

Example 5

The following ethylene discharge example will help to elucidate the above procedure. Choose a 10-in. (0.25-m) dia. outlet piping and an allowable 40% builtup back pressure ratio:

$A_2 = 79$ in.^{2} = 0.051 m^2

$\rho_o = 399$ kg/m^3

From Eq. 59:

$$P_{2c} = \frac{218}{0.051}\left(\frac{(2.037 \times 10^6)(3.4)}{399}\right)^{1/2}$$

$= 5.63 \times 10^5$ N/m^2 = 81.7 psia

Since $P_{2c} > P_b$, flow is choked at the pipe exit:

$\eta_2 = P_{2c}/P_o = 81.7$ psia/295 psia = 0.277

$P_1 = 0.40$ (280 psig) + 14.7 = 126.7 psia

Hence:

$\eta_1 = P_1/P_o = 126.7/295 = 0.430$

Substituting η_1, η_2, and ω into Eq. 58 yields:

$4f(L/D) = 0.73$

$L = [0.73(0.25\text{ m})]/[4(0.005)] = 9.3\text{ m} = 30\text{ ft}$

which is the maximum allowable outlet pipe run. For a given outlet pipe, Eq. 58 can be used to calculate P_1 and, hence, the pressure drop in the piping. Unfortunately, the pencil-and-paper method would involve a few iterations.

Note that the above example illustrates a constant-diameter (or one-segment) tailpipe only. If the vent line contains enlargements or sometimes contractions, Eq. 58 would have to be employed for each pipe segment by calculating backwards from the pipe outlet. The end of each pipe segment must also be checked for choked flow or a sonic condition.

FURTHER CONSIDERATIONS

Near and above critical-point fluids

The ω parameter as defined by Eq. 29 (flashing flow) is applicable when the thermodynamic reduced temperature is less than 0.9 or reduced pressure is less than 0.5. Above this temperature and pressure, the ω correlation would tend to underestimate the mass flux and, hence, overpredict the required vent area.

Ref. 10 extends the correlations to near the thermodynamic critical point. An alternative approach would be to carry out an isenthalpic flash calculation to 70% of the inlet pressure (0.7 P_o) and determine the ω parameter via:

$$\omega = \frac{\dfrac{v_1}{v_o} - 1}{\dfrac{P_o}{P_1} - 1} \qquad (60)$$

where v_1 is the specific volume at P_1 (= 0.7 P_o). Then, for the nozzle case, Figure 3 can be used to determine both the choking mass flux, as well as the choked flow pressure. This method can also be used for supercritical fluids as well, again via an isenthalpic flash calculation. Ref. 20 provides some useful charts for a supercritical fluid (or nonideal gas) which may be accurate enough for design purposes.

Multicomponent systems

So far, the ω method has been developed primarily for one-component flashing fluids and two-component nonflashing systems. Limited success has been reported for multicomponent mixtures exhibiting minor vapor-phase composition change during depressurization and flashing *(2,21)*. Pseudo one-component physical properties can be estimated based on liquid-phase composition, except for the heat of vaporization and vapor specific volume, which both should be based on vapor-phase composition.

In a recent study *(22)*, a total of 15 mixtures of industrial interest were evaluated at conditions varying from saturated liquid to two-phase, to almost all-vapor. Overall, the ω correlation appears adequate for reasonably narrow-boiling mixtures (those with a difference in nominal atmospheric-pressure boiling points between the lightest and heaviest components of less than 80°C). But for wide-boiling-range mixtures and for nonideal VLE systems, such as those systems containing hydrogen in particular, the ω correlation underpredicts the flow significantly. In fact, for most of the hydrogen-containing mixtures, the mass flux can be closely predicted based on the purely nonflashing flow correlation ($\omega = \alpha_o$).

However, even for the most nonideal systems examined, the simple expansion law, Eq. 28, can describe reasonably well the fluid behavior upon depressurization. Hence, it was suggested to evaluate the ω parameter based on the actual flash behavior to 90% of the inlet pressure (0.9 P_o), via Eq. 60 for nozzle flow (PRV) application.

Complex relief piping

It is quite common for relief devices to discharge to a main vent header and so the calculation of pressure drop in the relief piping can get tedious when multiple reliefs occur simultaneously. Essentially, one has to break down the relief configuration into constant-diameter segments and assign the corresponding flow rates and stream compositions. Typically, one would start at the exit of the vent header. By carrying out two flash calculations involving all the streams to two pressures — one to ambient (back) pressure and the other to a higher than ambient (back) pressure — a representative ω value can be obtained:

$$\omega = \frac{\dfrac{v_{22}}{v_{11}} - 1}{\dfrac{P_{11}}{P_{22}} - 1} \qquad (61)$$

Here one can use Eq. 59 to calculate the exit choking pressure if, indeed, it is choked when $P_o = P_{11}$ and $\rho_o = \rho_{11}$ (or $v_o = v_{11}$). Having established the vent header exit pressure, one can then employ Eq. 58 to calculate the pressure at the beginning of the pipe segment. Note that:

$$G^* = \frac{W/A_p}{\sqrt{P_o\rho_o}} \qquad (62)$$

is defined based on the total flow in the segment and the pipe flow area.

This method should be adequate for a low-pressure drop situation (about 10% of set pressure), in particular, when the relief devices are PRVs. For rupture disk installations (without a PRV as a flow-controlling device), the above method generally yields satisfactory results. However, for some high-pressure situations and especially for complex mixtures such as hybrid systems (*i.e.*, containing both flashing components and noncondensible gases), the two-parameter expansion law of Eq. 25 may be necessary.

By carrying out two flash calculations at, say, 0.9 P_o and 0.5 P_o, the a and b parameters of Eq. 25 can be found easily. The treatment of nozzle flow and pipe flow follows closely the development presented here, although the actual numerical calculation is most expediently performed by computer. Rather than seeking the analytical closed-form solution for the integral momentum equation based on Eq. 26, the integration can be efficiently carried out numerically.

Subcooled liquid inlet

Flashing discharge of an inlet containing subcooled liquid requires special treatment, and the resulting flow rate can be substantially higher than the saturated liquid inlet case at the same inlet pressure. In this treatment, the first term in the ω parameter vanishes and a "saturated" ω parameter can be defined as:

$$\omega_s = \rho_{fo} C_p T_o P_s \left(\frac{v_{fgo}}{h_{fgo}} \right)^2 \qquad (63)$$

where P_s is the saturation vapor pressure corresponding to inlet temperature T_o. The two-phase expansion law now takes the form:

$$\frac{v}{v_o} = \omega_s \left(\frac{P_s}{P} - 1 \right) + 1 \qquad (64)$$

The generalized solutions for nozzle discharge *(23)* and pipe discharge *(24)* are divided into low and high subcooling regions, delineated by a "transition" saturation pressure ratio (P_{st}/P_o):

$$\eta_{st} \equiv \frac{2\omega_s}{1 + 4f\dfrac{L}{D} + 2\omega_s} \qquad (65)$$

If the inlet subcooling is high such that $\eta_s < \eta_{st}$ (or $P_s < \eta_{st} P_o$), then the above methodology simply yields a critical mass flux given by:

$$G_c = \left(\frac{2\rho_{fo}(P_o - P_s)}{1 + 4f\dfrac{L}{D}} \right)^{0.5} \qquad (66)$$

Note that this expression differs from the classical incompressible pipe flow formula in that the pressure drop is not the total available ($P_o - P_b$), but rather is due to choking condition at the exit ($P_o - P_s$). Figure 13 is a design chart for the nozzle flow. Consult Refs. 23 and 24 for development of additional equations.

Hybrid mixed flows

A flashing discharge in the presence of a noncondensible gas can be considered to be a hybrid two-phase flow system. In this system, the ω method describes an expansion law separately in terms of the condensible vapor (denoted by subscript v here) and noncondensible gas partial pressure (denoted by subscript g):

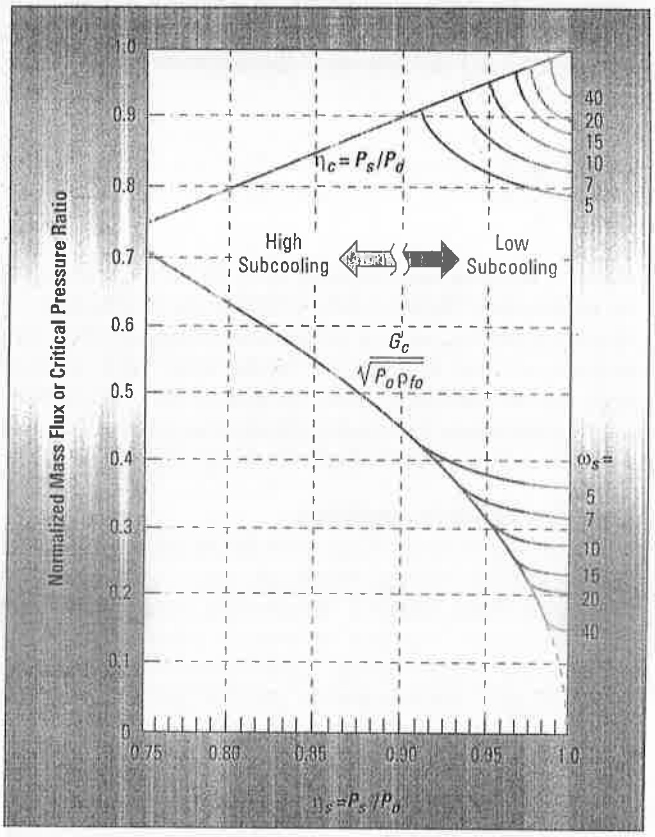
$$\frac{v}{v_o} = \omega \left(\frac{P_{vo}}{P_v} - 1 \right) + 1 \qquad (67)$$

$$\frac{v}{v_o} = \alpha_o \left(\frac{P_{go}}{P_g} - 1 \right) + 1 \qquad (68)$$

Here, ω is defined in terms of the vapor (flashing) component, similar to the subcooled inlet case:

$$\omega = \alpha_o + (1 - \alpha_o) \rho_{fo} C_p T_o P_{vo} \left(\frac{v_{fgo}}{h_{fgo}} \right)^2 \qquad (69a)$$

or



■ *Figure 13. Nozzle critical flow of inlet subcooled liquid.*

$$\omega = \alpha_o + (1 - \alpha_o)\,\omega_s \qquad (69b)$$

Figure 14 illustrates the graphical solution of hybrid flow through nozzles at a fixed $\omega_s = 10$, but with varying degrees of void fraction and inlet-gas mole-fraction P_{go}/P_o. At a given α_o, this chart illustrates the effect of the gas partial pressure in flow augmentation, which is particularly drastic at a low void fraction. In the limit when $\alpha_o = 0$, the liquid inlet case, the result reduces to the inlet subcooled liquid solution. Interested readers are referred to Refs. 15 and 16 for detailed solutions of both the nozzle discharge case and the pipe discharge case.

An alternative method may be to use the one-parameter expansion law (requiring one flash calculation) or the two-parameter expansion law (requiring two flash calculations).

Nonequilibrium effect in short nozzles

In considering relieving loads to downstream disposal equipment and source terms in atmospheric dispersion, it may be prudent to assess the effect of thermal nonequilibrium and the consequences of a much higher discharge flow. Typically a nonequilibrium flow condition (superheated liquid) exists for a flashing system if the nozzle/duct length is less than 0.1 m (4 in.). For a bounding estimate, we can use the assumption of nonflashing flow and estimate the mass-flow rate based at $\omega = \alpha_o$ simply from Figure 3.

For those PRV nozzles with a nearly constant diameter section, one can use the empirical correlation developed by Fauske *(25)* where:

$$G = \frac{G_E}{\sqrt{N_{NE}}} \qquad (70)$$

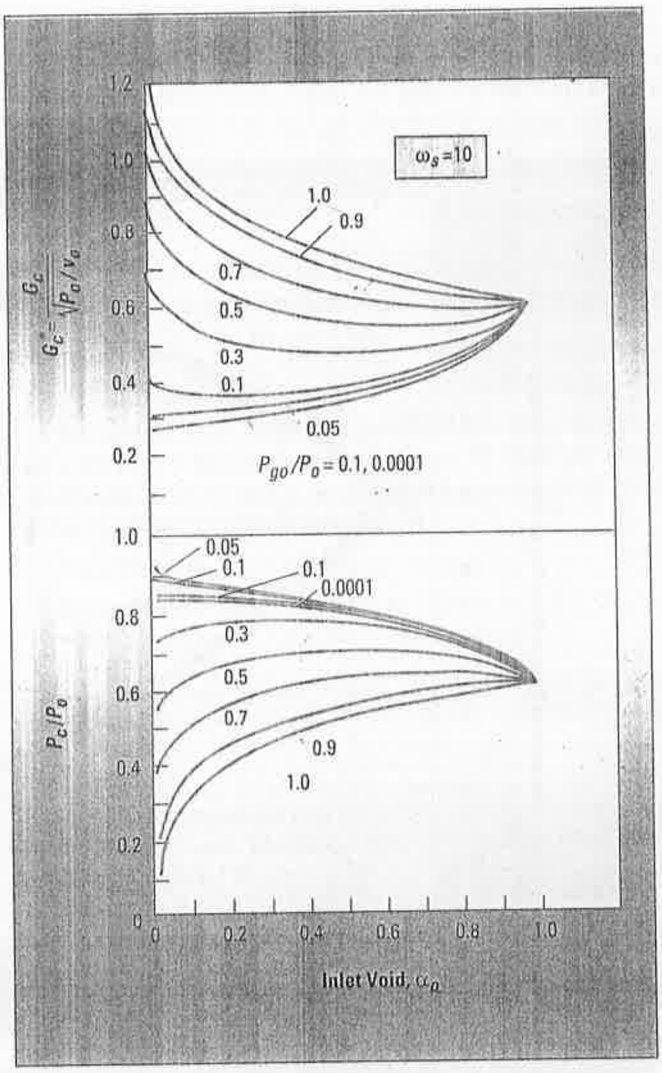
where G_E denotes the equilibrium mass flux, while a nonequilibrium parameter is given in terms of the constant-diameter section length L:

$$N_{NE} = \left(\frac{G_E}{G_{NE}}\right)^2 + \frac{L}{L_E} \qquad (71)$$

Here, L_E the equilibrium flow length is taken to be 0.1 m and G_{NE} is the nonflashing (nonequilibrium) mass flux. Equation 70 provides, for the time being, a practical approach for estimating the mass flux in nozzles with a constant-diameter section varying from 0–0.1 m.

Update on ω parameter

The original ω development for flashing flow is based on Eq. 29a. A total of 11 widely different fluids were successfully correlated in terms of the critical mass flux and the critical flow pressure ratio (see Figure 15). However, these data for G and P_c/P_o, which are based on HEM calculations, show an increasing deviation from the theoretical curves, Eqs. 31 and 32, as ω goes below 2, *i.e.*, at the high



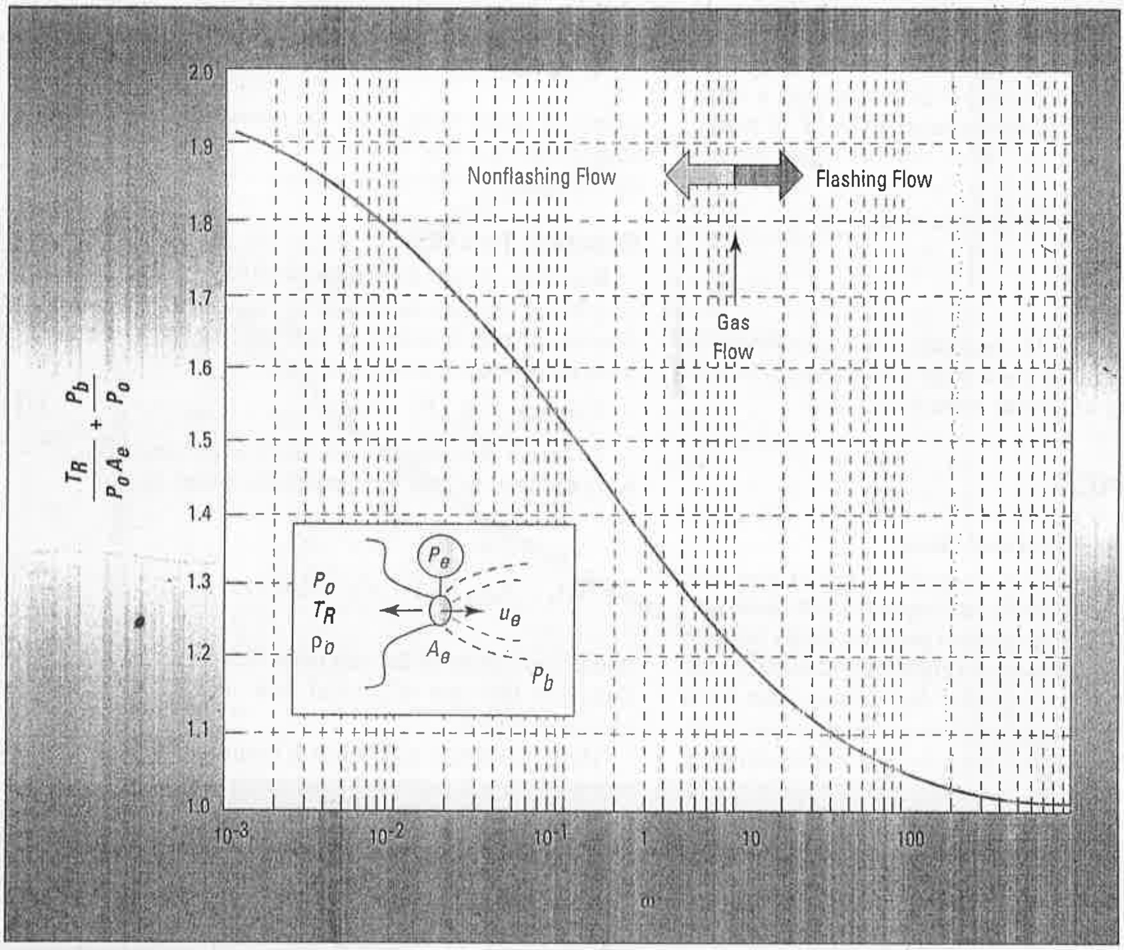
■ *Figure 14, Typical chart for flashing flow with a noncondensible gas through a nozzle.*

quality end. This deviation is due to the difference between the assumed isenthalpic and the actual isentropic expansion processes.

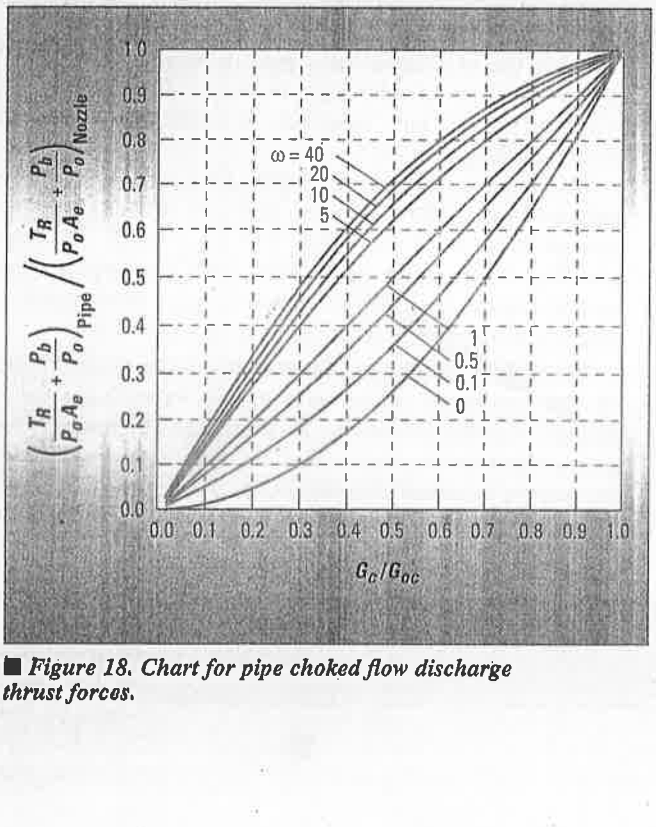
The maximum deviation at $\omega = 1$ (near all-vapor flow) is about 10%. Recent analysis of the sonic two-phase flow velocity *(26)* suggests a slightly modified form of the ω expression as given in Eq. 29c.

The extra term is $(1 - 2\,P_o v_{fgo}/h_{fgo})$ which is always less than unity.

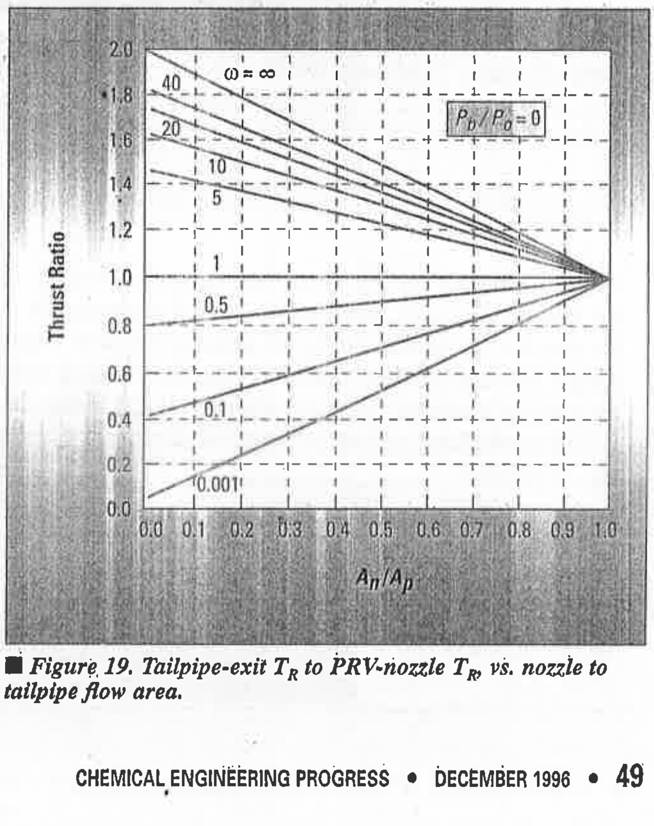
For all-vapor flow, ω takes on a value slightly less than 1.0. The success of such an ω can be demonstrated in Figure 16 where published theoretical HEM choked flow data for steam-water systems are in close agreement with the ω solutions. Very little deviation is noted as the inlet quality x_o approaches unity (all steam). This, therefore, represents an improvement over the earlier empirical curve fit in the region when $\omega < 3$ *(10)*.



■ *Figure 17. Generalized chart for choked-flow nozzle discharge thrust forces.*



■ *Figure 18. Chart for pipe choked flow discharge thrust forces.*



■ *Figure 19. Tailpipe-exit T_R to PRV-nozzle T_R, vs. nozzle to tailpipe flow area.*

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Note that the thrust from a PRV nozzle can be easily found from the nozzle chart of Figure 17. The results are presented in Figure 19 for a back pressure to set pressure ratio P_b/P_s of 0 (consult *(7)* for charts at 0.1, 0.2, and 0.5 ratios). This chart should provide a conservative design. Since the nozzle and the tailpipe carry the same mass flow, *i.e.*, $W = A_n G_o = A_p G$, the thrust ratio is shown as a function of either A_n/A_p (nozzle area to tailpipe area) or G/G_o (tailpipe mass flux to nozzle mass flux). Note that the thrust at the tailpipe exit is higher than for the nozzle case for the flashing flow ($\omega > 1$) even though the mass flow is the same. The increase is due to a higher exit velocity as a result of flashing. However, the maximum augmentation in thrust is only a factor of two. For vapor or gas discharge, the thrust ratio is identically unity. For nonflashing discharge, the thrust ratio is less than unity, implying that the thrust at the tailpipe exit is less than the nozzle discharge. CEP

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